Unit-3
Transient Circuit Analysis

Syllabus

Transient Circuit Analysis: Pre- Requisites: Laplace Transform& Concept of Initial conditions. Natural response and forced response, Transient behaviour of RL, RC and RLC networks, Evaluation of initial conditions, Transform Impedance, Transient response and steady state response for arbitrary inputs (DC and AC), Evaluation of time response of RL, RC and RLC networks with and without initial conditions both through classical and Laplace transform methods.

Course Outcome

Analyse steady-state responses and transient response of DC and AC circuits using classical and Laplace transform methods.

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UNIT-III

TRANSIENTS

Transients are present in the ckt when the ckt is subjected to any changes either by changing the source magnitude or any ckt element provided ckt consist of energy storage elements like Inductor & Capacitor bcz. inductor doesn't allow the sudden change in current & Capacitor doesn't allow the sudden change in voltage.

$t=0^-$ time instant just before switch operation (S.S before the switch operation)

$t=0$ exact instant of switch operation.

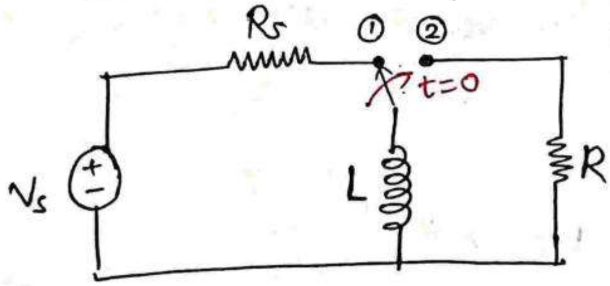
$t=0^+$ time instant just after the switch operation

$t=\infty$ Steady state after switch operation.

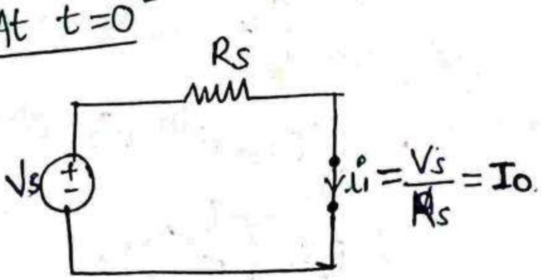
Note : $i_L(0^-) = i_L(0) = i_L(0^+)$

$V_C(0^-) = V_C(0) = V_C(0^+)$

Source free RL ckt :

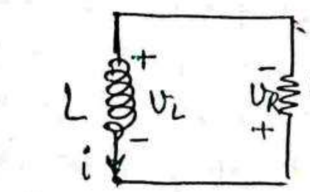


At $t=0^-$



$i(0^-) = i(0) = i(0^+) = I_0$

for $t>0$



KVL:

$$v_L + v_R = 0$$

$$L\frac{di}{dt} + iR = 0$$

$$L\frac{di}{dt} = -iR$$

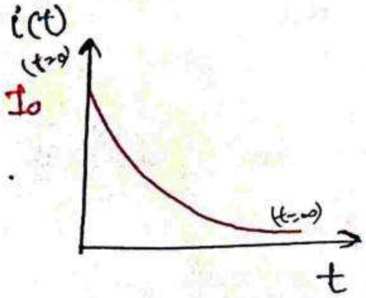
$$\int_{I_0}^{i(t)} \frac{1}{i}\, di = \int_0^t -\frac{R}{L} dt \Rightarrow \boxed{i(t) = I_0 e^{-\frac{R}{L}t}}$$

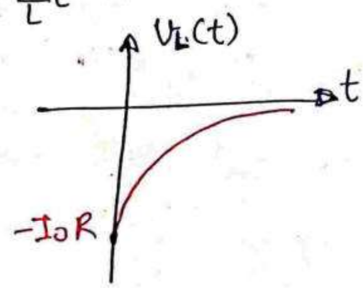
- $v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(I_0 e^{-\frac{R}{L}t}\right) = L\left(-\frac{R}{L}\right) I_0 e^{-\frac{R}{L}t}$

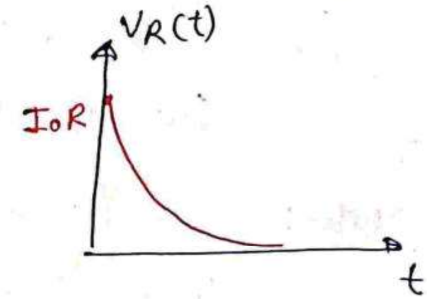
$$\boxed{v_L = -I_0 R e^{-\frac{R}{L}t}}$$

$\tau = \frac{L}{R}$

$$v_R(t) = I_0 R e^{-\frac{R}{L}t}$$







★ In the discharging inductor, the current direction does not changed but polarity of voltage across the inductor gets reversed.

Energy consideration

$$w_L(t) = \frac{1}{2} L i^2(t)$$
$$= \frac{1}{2} L \left[I_0 e^{-t/\tau}\right]^2$$
$$= \frac{1}{2} L I_0^2 e^{-2t/\tau}$$

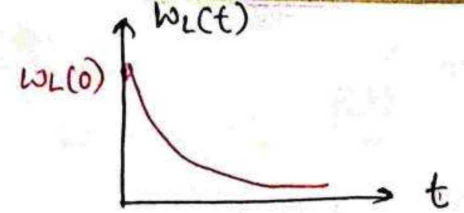
$$w_L(0) = \frac{1}{2} L I_0^2$$

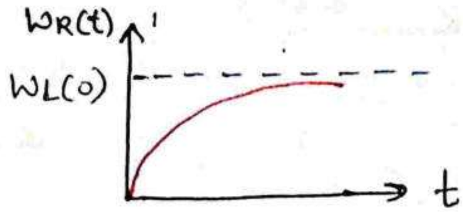
$$\boxed{w_L(t) = w_L(0) e^{-2t/\tau}}$$

Energy stored by Inductor

$$w_R(t) = \int_0^t p(t)\, dt$$
$$= \int_0^t v_R(t) \cdot i_R(t)\, dt$$
$$= \int_0^t (I_0 R e^{-t/\tau}) \cdot (I_0 e^{-t/\tau})\, dt$$
$$= I_0 R \int_0^t e^{-2t/\tau} dt$$
$$w_R(t) = \frac{1}{2} L I_0^2 \left[1 - e^{-2t/\tau}\right]$$

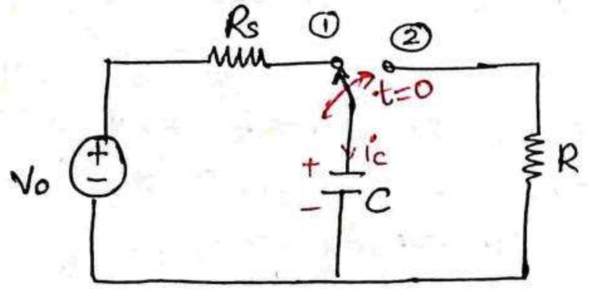
Energy dissipated in Resistor →
$$\boxed{w_R(t) = w_L(0)\left[1 - e^{-2t/\tau}\right]}$$



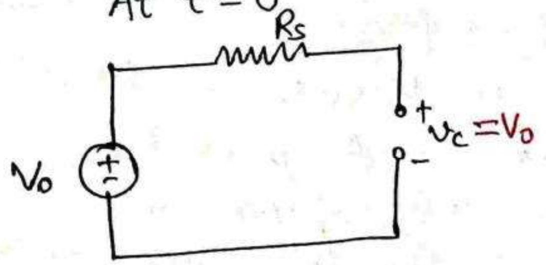


∘ At $t=0^-$ Analysis

Source Free RC Ckt –

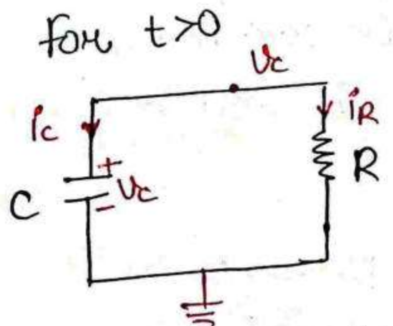


At $t=0^-$



$$v_c(0^-) = v_c(0) = v_c(0^+) = V_0$$

for $t>0$



$$i_c + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v_c}{R} = 0$$

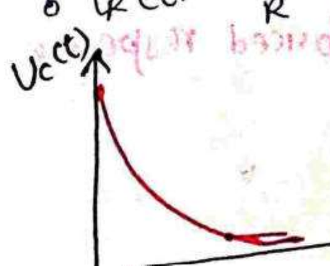
$$C\frac{dv}{dt} = -\frac{v_c}{R}$$

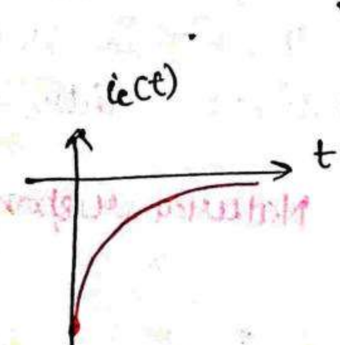
$$\int_{V_0}^{v_c(t)} \frac{1}{v_c}\,dv = \int_0^t -\frac{1}{RC}\,dt \Rightarrow \boxed{v_c(t) = V_0 e^{-\frac{t}{RC}}}$$

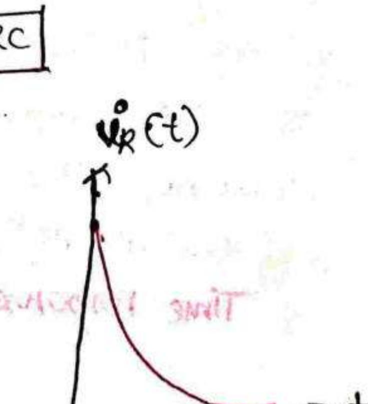
$e^{-t/\tau}$ $\boxed{\tau = RC}$

$$\boxed{i_c(t) = -\frac{V_0}{R} e^{-t/RC}}$$

∘ $i_R(t) = \frac{V_0}{R} e^{-t/RC}$







Energy consideration

$$W_C(t) = \frac{1}{2} C V^2(t)$$
$$= \frac{1}{2} C \left[V_0 e^{-\frac{t}{RC}} \right]^2$$
$$= \frac{1}{2} C V_0^2 e^{-\frac{t}{RC}}$$
$$W_C(0) = \frac{1}{2} C V^2$$
$$\boxed{W_C(t) = W_C(0) e^{-\frac{2t}{\tau}}}$$

$$W_R(t) = \int_0^t p(t)\,dt$$
$$= \int_0^t V_R(t) \cdot i_R(t)\,dt$$
$$= \int_0^t$$
$$W_R(t) = \frac{1}{2} C V^2 \left[1 - e^{-2t/\tau} \right].$$
$$\boxed{W_R(t) = W_C(0) \left[1 - e^{-2t/\tau} \right]}$$

TIME constant –

- It is the time taken by the response to reach 36.7% of its initial value while discharging or decaying or it is the time taken by the response to reach 36.2% of its final value while charging.
- An electrical n/w has 2 types of responses.

(1) The response of a n/w without a source in it is called as the **Natural Response** & it gives the transient response. This response only depends on the nature of passive elements & it is Independent of the type of the i/P. This response is obtained by solving the complementary function in the differential eq^n.

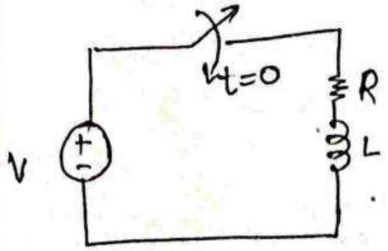
(2) Response of a n/w with a source present in it is called as Forced response & it leads to steady state response. This response is independent of the nature of the passive elements & depends only on the type of i/P. This response is obtained by solving the particular integral part in the differential eq^n.

Time response = Natural response + Forced response

Natural response → ZIR → tr → CF

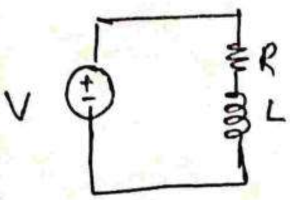
Forced response → ZSR → S.S → PI

RL ckt with Source –



$$i(0^-) = i(0) = i(0^+) = 0$$

For $t > 0$



$$V = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$$i(t) = i_{CF} + i_{PI}$$

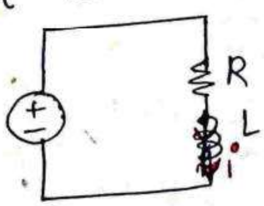
i) CF.

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$i_{CF} = Ae^{-\frac{R}{L}t}$$

ii) PI.

At $t = \infty$



$$i_{PI} = \frac{V}{R}$$

$$i(t) = Ae^{-\frac{R}{L}t} + \frac{V}{R}$$

At $t=0$, $i=0$

$$0 = A.1 + \frac{V}{R}$$

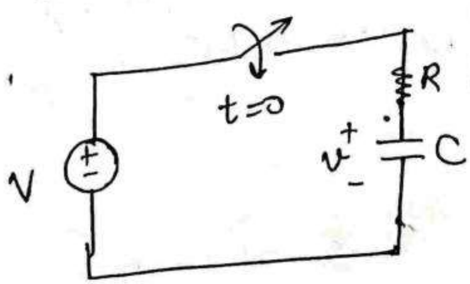
$$A = \underset{i(0^+)}{0} - \underset{i(\infty)}{\frac{V}{R}}$$

$$i(t) = \frac{V}{R} + \left(0 - \frac{V}{R}\right)e^{-\frac{R}{L}(t)}$$

$$\boxed{i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right]e^{-t/\tau}}$$

dc & $Au(t)$ step functn.   RL & RC

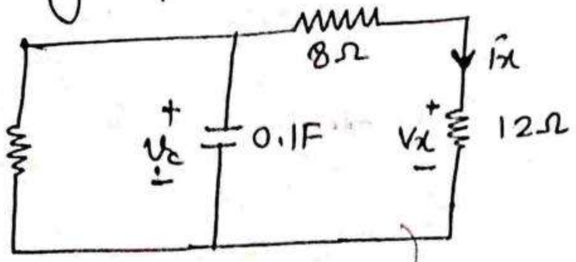
RC ckt with Source –



$$\boxed{V_C(t) = V_C(\infty) + \left[V_C(0^+) - V_C(\infty)\right]e^{-t/\tau}}$$

dc & $Au(t)$ RL & RC

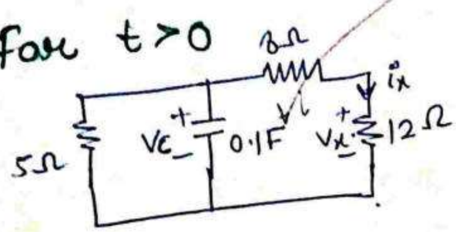
Q. find $v_c(t)$, $v_x(t)$ & $i_x(t)$ for $t>0$ when initial voltage of capacitor is 15 volts



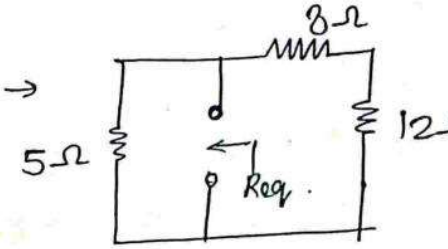
without source

① $V_c(0^-) = V_c(0^+) = 15$

② for $t>0$



③ $v_c(t) = V_0 e^{-t/\tau}$ ← (without source) ⟹ Req →



$V_0 = 15V$

$\tau = \underline{Req}\cdot C$

$Req = 5 \,||\, (8+12)$

$Req. = 4\Omega$

$\tau = (4)(0.1)$

$\boxed{\tau = 0.4 \text{ sec}}$

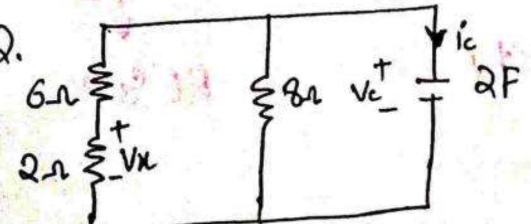
$\boxed{v_c(t) = 15 e^{-t/0.4} V}$

• $v_x(t) = \dfrac{v_c(t)}{8+12} \times 12$ ← [voltage division rule] $= \dfrac{15}{20} e^{-t/0.4} \times 12$

$\boxed{v_x(t) = 9 e^{-t/0.4} V}$

• $i_x(t) = \dfrac{v_c(t)}{8+12} = \dfrac{15}{20} e^{-t/0.4} \Rightarrow \boxed{i_x(t) = 0.75 e^{-t/0.4} A}$

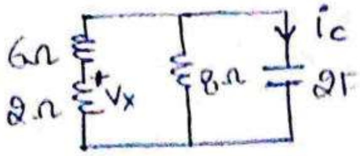
Q.



$\left.\begin{matrix} v_c(t) \\ v_x(t) \\ i_c(t) \end{matrix}\right\} = ?$ $\quad V_c(0) = 3V$

$\rightarrow V_C(0^-) = V_C(0^+) = 3$ initial condition

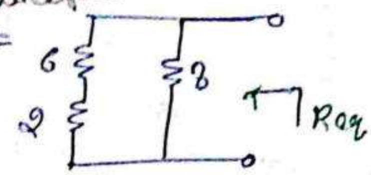
$\rightarrow$ for $t > 0$



- $V_C(t) = V_0 e^{-t/\tau}$ type of source / sourcefree

$V_0 = 3\text{ V}$

Req. =



$\rightarrow \tau = R_{eq} \cdot C.$

$\underline{\underline{R_{eq}}} = 4\,\Omega.$

$\tau = (4)(2)$

$$\boxed{V_C(t) = 3e^{-t/8}\text{ V}}$$

- $\cancel{i_c(t) = 2\frac{d}{d}}$ $\quad i_c = C\frac{dV_c}{dt}.$

$i_c(t) = 2\frac{d}{dt}\left(3e^{-t/8}\right)$

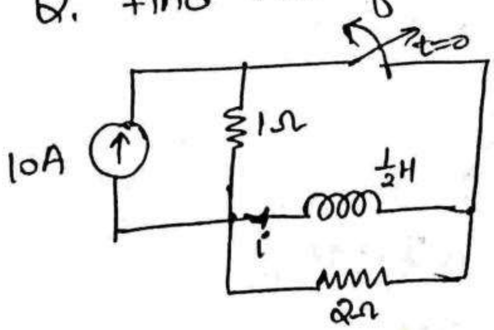
$= 2(3)\left(-\frac{1}{8}\right)e^{-t/8}$

$$\boxed{i_c(t) = -\frac{3}{4}e^{-t/8}\text{ A}}$$

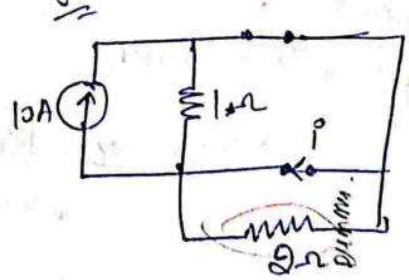
- $V_x(t) = \dfrac{V_c(t)}{6+2} \times 2 = \dfrac{3}{8}e^{-t/8} \times 2$

$$\boxed{V_x(t) = \frac{3}{4}e^{-t/8}\text{ V}}$$

Q. Find $i(t)$ for $t > 0$. with switch

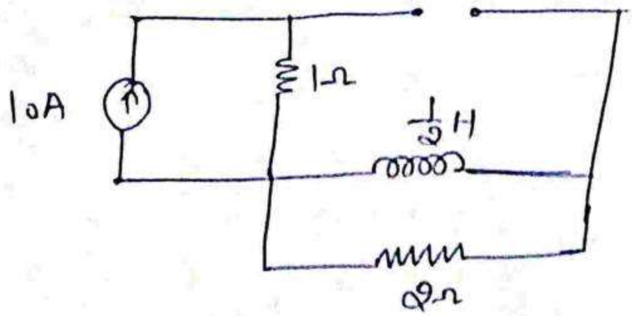


Sol^n At $t = 0^-$

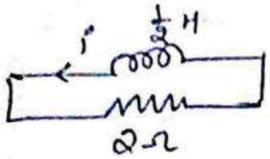


initial condition $\rightarrow i(0^-) = i(0^+) = 10\text{ A}$

for $t>0$.



with source
↓
kar load ki tarah kaar karaye



$$i(t) = I_0 e^{-t/\tau}$$

$$I_0 = 10\,A$$

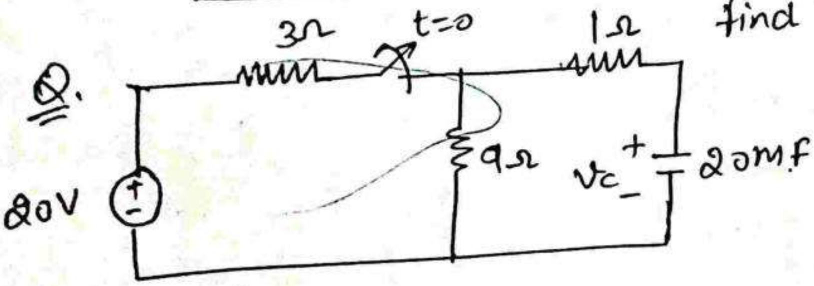
$$\tau = \frac{L}{R}$$

$$\tau = \frac{1/2}{2} = \frac{1}{4}\ \text{sec.}$$

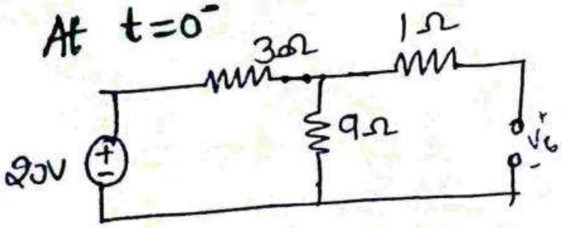
$$\boxed{i(t) = 10e^{-4t}\ A}$$

find $v_c(t)$ for $t>0$

Q.



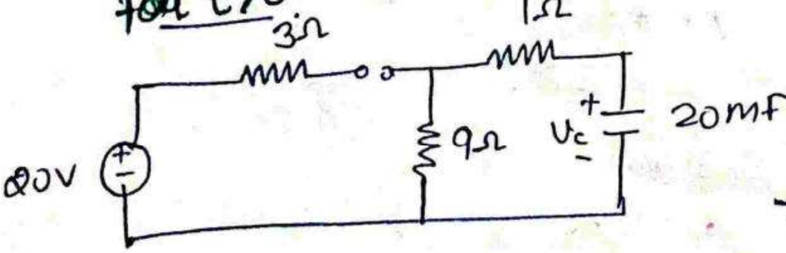
At $t=0^-$



$$v_c = \frac{20}{3+9} \times 9$$

$$v_c(0^-) = v_c(0^+) = 15\,V.$$

For $t>0$



$$V_c(t) = V_0 e^{-t/\tau}$$

$$V_0 = 15\,V$$

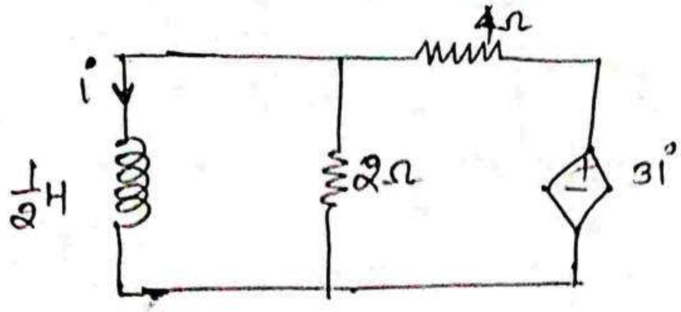
$$\tau = R_{eq} \cdot C$$

$$= (9+1)(20m)$$

$$= 0.2\ \text{sec.}$$

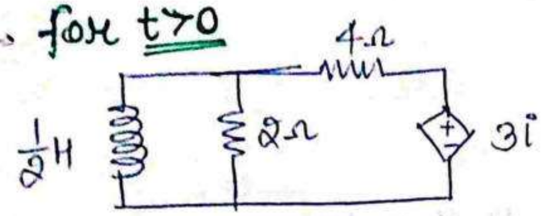
$$\boxed{v_c(t) = 15e^{-t/0.2}\ V}$$

Q. In the below given electrical n/w. find the current i(t) when initial current through inductor is 10A.



→ $i(0^-) = i(0^+) = 10$

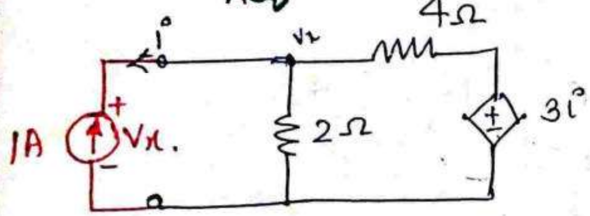
→ for t>0



$i(t) = I_0 e^{-t/\tau}$ [without source]

$I_0 = 10$ A

$\tau = \frac{L}{R_{eq}}$ $R_{eq} = R_{TH}$



$-1 + \frac{V_x}{2} + \frac{V_x - 3i}{4} = 0$

As $i = -1$

$-1 + \frac{3}{4}V_x + \frac{3}{4} = 0$

$\frac{3}{4}V_x = \frac{1}{4}$

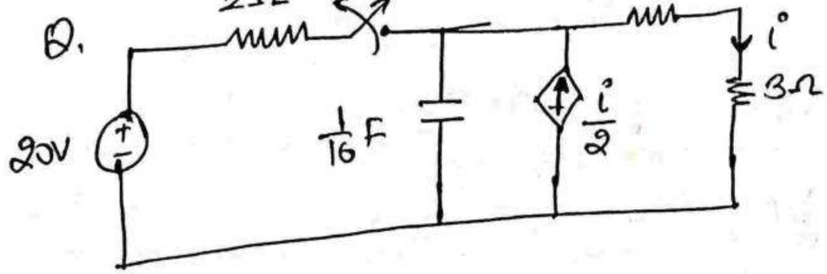
$V_x = \frac{1}{3}$

$R_{eq} = \frac{V_x}{1} = \frac{1}{3}\,\Omega$

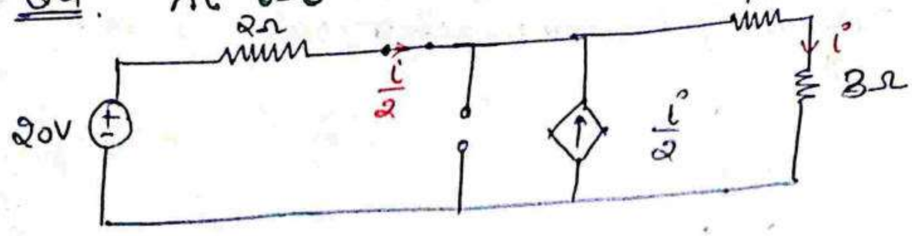
$\tau = \frac{L}{R_{eq}} = \frac{3}{2}$ sec.

$\boxed{i(t) = 10e^{-\frac{2}{3}t}\ \text{A}}$

Q. In the below given elect. n/w. find current i(t) for t>0



Sol^n. At $t=0^-$



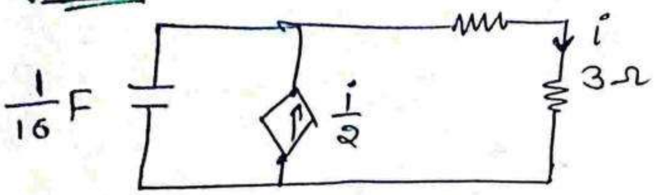
$$-20 + 2\left(\frac{i}{2}\right) + 4i = 0$$

$$5i = 20 \Rightarrow i = 4A$$

$$v_c = 4i = 4(4) = 16$$

$$v_c(0^-) = v_c(0^+) = 16V$$

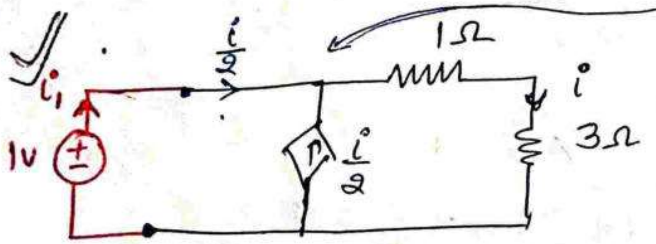
For $t>0$.



$$V_c(t) = V_0 e^{-t/\tau}$$

$$V_0 = 16V$$

$$\tau = R_{eq} C.$$



$$i = \frac{1}{1+3} = \frac{1}{4} A$$

$$i_1 = \frac{i}{2}$$

$$i_1 = \frac{1}{8} A$$

$$R_{eq} = \frac{1}{i_1} = \frac{1}{\frac{1}{8}} = 8\Omega$$

$$\tau = R_{eq}.C$$

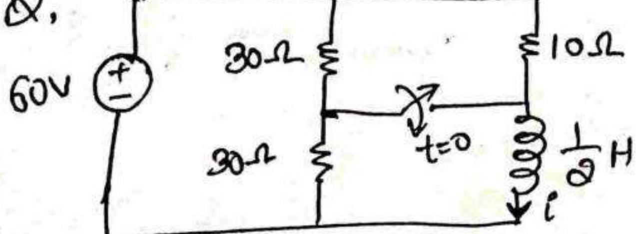
$$= (8)\left(\frac{1}{16}\right) = \frac{1}{2}\ sec.$$

$$v_c(t) = \cancel{V_c e^{t}}\ 16 e^{-2t} V$$

$$i(t) = \frac{v_c(t)}{1+3} = \frac{16}{4} e^{-2t}$$

$$i(t) = 4e^{-2t} A \quad \underline{t>0}.$$

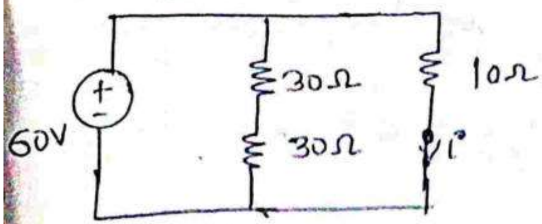
Q.



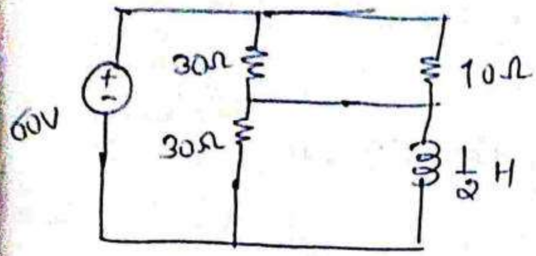
find $i(t)$.

At $t=0^-$

$$i(0^-) = i(0^+) = 6A = 7A$$



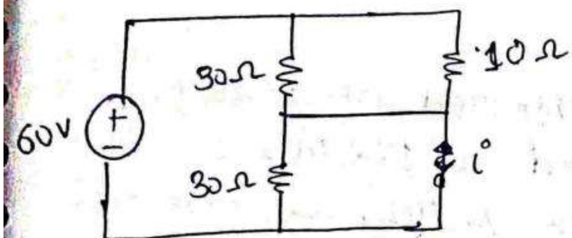
for $t>0$. with Source



$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

$$i(0^+) = 6$$

At $t=\infty$. (short Inductor)

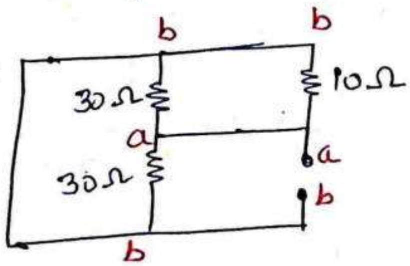


$$i = \frac{60}{30 \| 10} = \frac{60}{30 \times 10} \times 40 = 8\,A$$

$$i(\infty) = 8\,A$$

• $\tau = \dfrac{L}{R_{eq}}$

Req



$$R_{eq} = 30 \| 30 \| 10$$

$$= 15 \| 10$$

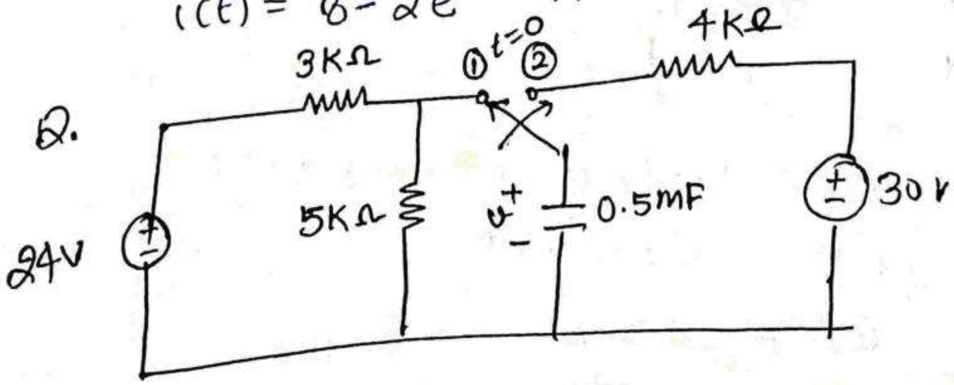
$$R_{eq} = 6\,\Omega.$$

$$\tau = \frac{1/2}{6} = \frac{1}{12}\text{ sec.}$$

$$i(t) = 8 + [6-8]e^{-12t}$$

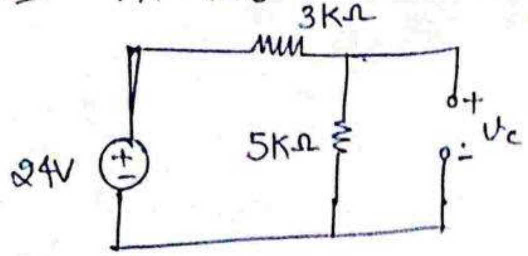
$$i(t) = 8 - 2e^{-12t}\text{ A.}$$

Q.



Find $V_c(t)$.

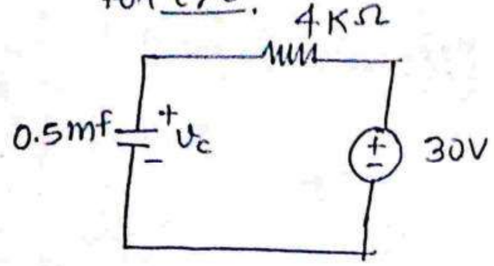
Sol^n At $t=0^-$



$$v_c = \frac{24}{3k+5k} \cdot 5K$$

$$v_c(0^-) = v_c(0^+) = 15\,V$$

For $t>0$.



$$v_c(t) = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$$

$$v_c(\infty) = 30\,V$$

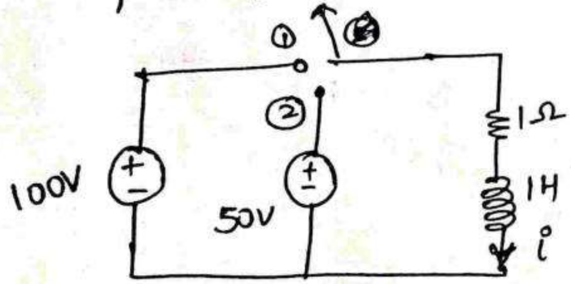
$$\tau = RC = (4K)(0.5m)$$

$$\tau = 2\text{ sec.}$$

$$v_c(t) = 30 + [15-30]e^{-t/2}$$

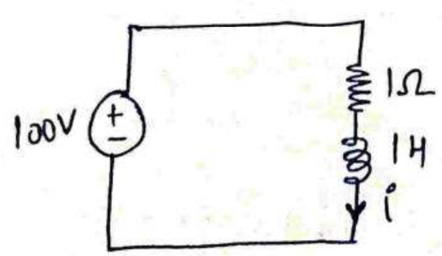
$$v_c(t) = 30 - 15e^{-0.5t}\ V.$$

Q. At $t=0$ sec, the switch S is connected at position 1 & after 1 time constant it is moved to position-2. find the current response $i(t)$ when switch is connected at position-2.



Sol^n $i(0^-) = i(0^+) = 0$

for $0 \le t \le 1$



$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

$$i(\infty) = \frac{100}{1} = 100\,A$$

$$\tau = \frac{L}{R} = \frac{1}{1} = 1\text{ sec.}$$

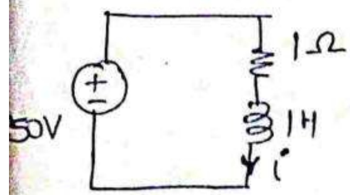
$$i(t) = 100[1-e^{-t}] \quad 0 \le t \le 1$$

$$i(1) = 100[1-e^{-1}]$$

$$i(1) = 63.2\,A$$

$i(1^-) = i(1) = i(1^+) = 63.2\,A$

For $t \geq 1$



$$i(t) = i(\infty) + [i(1^+) - i(\infty)]e^{-(t-1)/\tau}$$

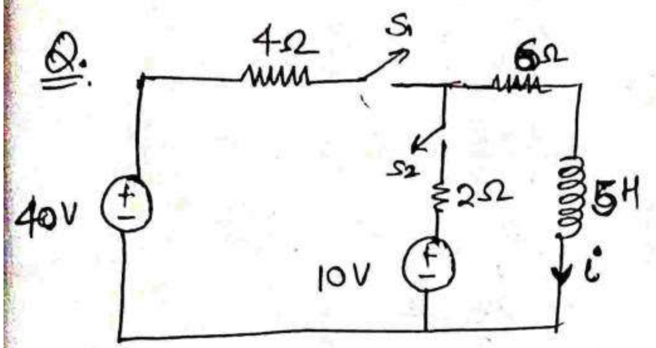
$$i(\infty) = \frac{50}{1} = 50\,A$$

$$\tau = \frac{L}{R} = 1\ sec.$$

$$i(t) = 50 + [63.2 - 50]e^{-(t-1)}$$

$$i(t) = 50 + 13.2\,e^{-(t-1)}\ A.$$

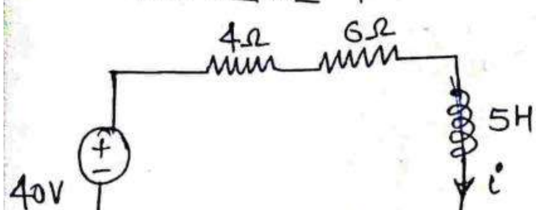
Q.



At $t=0$ sec. Switch S_1 is closed & 4 sec later switch S_2 is closed. Calculate the current response $i(t)$ for $t>0$.

Soln $i(0^-) = i(0^+) = 0$

For $0 \leq t \leq 4$



$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

$$i(\infty) = 4\,A$$

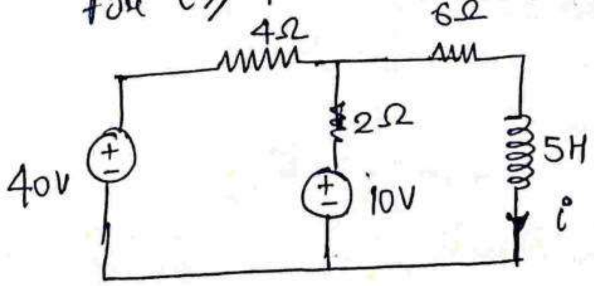
$$\tau = \frac{L}{R} = \frac{5}{10} = \frac{1}{2}\ sec.$$

$$i(t) = 4[1 - e^{-2t}] \quad 0 \leq t \leq 4.$$

$$i(4) = 4[1 - e^{-8}] \approx 4$$

$$i(4^-) = i(4) = i(4^+) = 4\,A$$

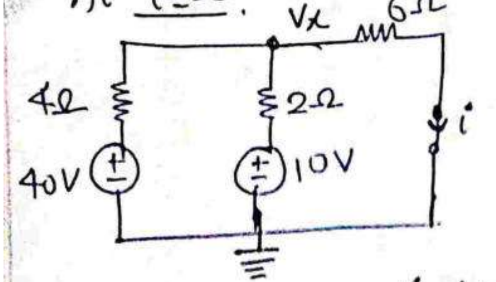
For $t \geq 4$



$$i(t) = i(\infty) + [i(4^+) - i(\infty)]e^{-(t-4)/\tau}$$

~~At t=∞~~

At $\underline{t=\infty}$.



$$\frac{V_x-40}{4}+\frac{V_x-10}{2}+\frac{V_x}{6}=0$$

$$3V_x-120+6V_x-60+2V_x=0$$

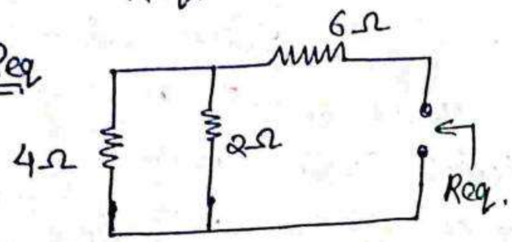
$$V_x=\frac{180}{11}\,V$$

$$i=\frac{V_x}{6}=\frac{30}{11}$$

$$i(\infty)=2.727\text{ A}.$$

$$\tau=\frac{L}{R_{eq}}$$

Req



$$R_{eq}=[4\|2]+6$$

$$=\frac{4\times 2}{4+2}+6$$

$$=\frac{4}{3}+6$$

$$=\frac{22}{3}\,\Omega$$

$$\tau=\frac{5}{22/3}$$

$$\tau=\frac{15}{22}\text{ sec.}$$

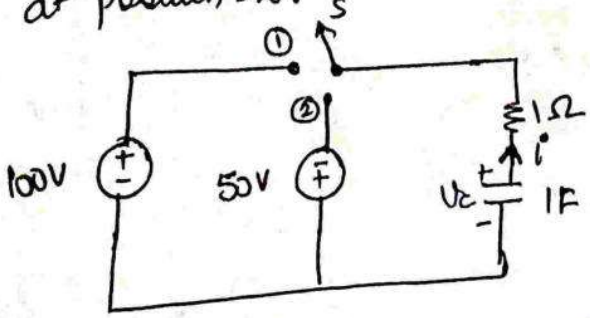
$$i(t)=2.727+[4-2.727]e^{-\frac{22}{15}(t-4)}$$

$$i(t)=2.727+1.273\,e^{-1.4667(t-4)}$$

$$i(t)=4[1-e^{-2t}] \qquad 0\le t\le 4$$

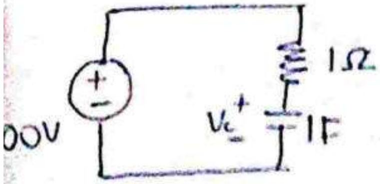
$$2.727+1.273\,e^{-1.4667(t-4)} \qquad t\ge 4$$

Q. At t=0 sec. the switch S is connected at position-1 & it is moved to position-2. Find after 1 time constant the current response i(t) when the switch is connected at position-2.



$v_c(0^-) = v_c(0^+) = 0\,V$

For $0 < t < 1$



$$v_c(t) = V_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$$

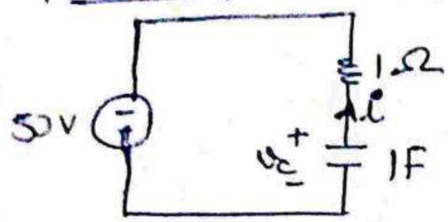
$$v_c(\infty) = 100V \quad , \quad \tau = 1\,sec.$$

$$v_c(t) = 100[1 - e^{-t}] \qquad 0 < t < 1$$

$$v_c(1) = 100[1 - e^{-1}] = 63.2\,V$$

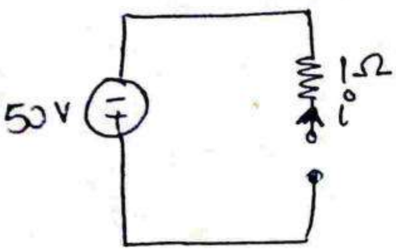
$$v_c(1^-) = v_c(1) = v_c(1^+) = 63.2\,V.$$

For $t > 1$.



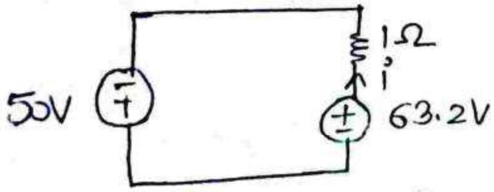
$$i(t) = i(\infty) + [i(1^+) - i(\infty)]e^{-(t-1)/\tau}$$

At $t = \infty$.



$$i(\infty) = 0\,A$$

At $t = 1^+$.

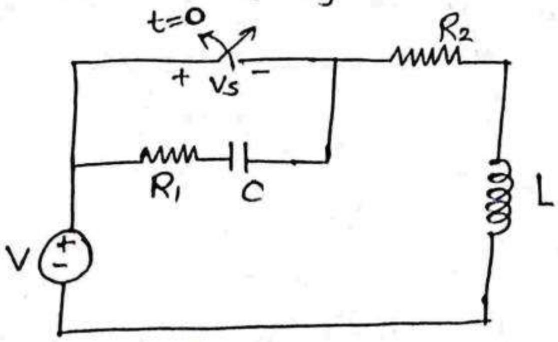


$$\boxed{i(t) = 113.2\,e^{-(t-1)}\,A}$$

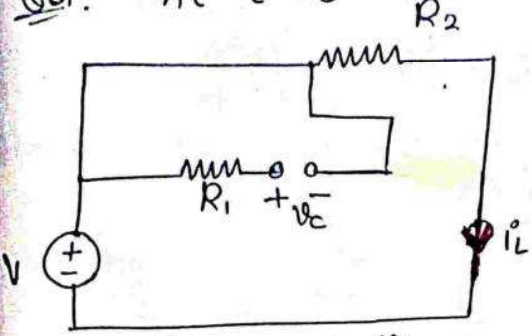
$$i = \frac{50 + 63.2}{1}$$

$$i = 113.2\,.$$

Q. Find the voltage across the switch V_s at $t=0^+$.



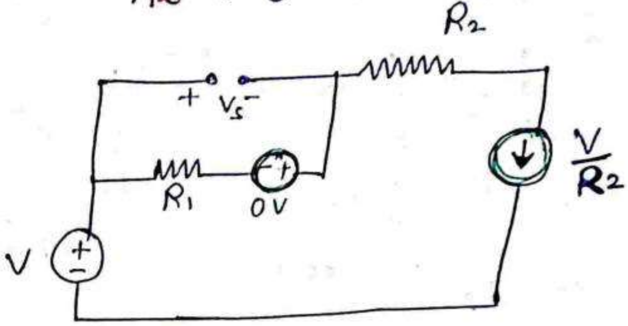
Sol^n. At $t=0^-$



$v_C(0^-) = v_C(0^+) = 0$

$i_L(0^-) = i_L(0^+) = \frac{V}{R_2}$

At $t=0^+$

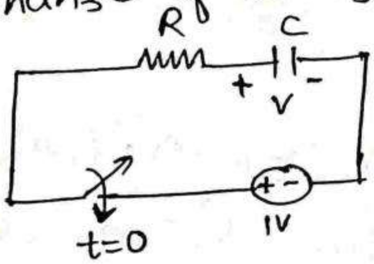


$V_R = \frac{V}{R_2} \cdot R_1$

As $V_S = V_R$

$V_S = V \cdot \frac{R_1}{R_2}$

Q. In the below given elect. n/w, find the time rate of change of voltage V at $t=0^+$.



Method-I.

$V(t) = 1 - e^{-t/RC}$

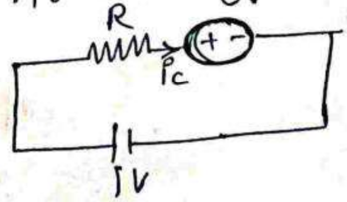
$\frac{dv}{dt} = \frac{1}{RC} e^{-t/RC}$

$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{1}{RC}$

Method-II.

$V(0^-) = V(0^+) = 0$

At $t=0^+$



$i_c(0^+) = \frac{1}{R}$

$i_c(0^+) = C\frac{dv}{dt}$

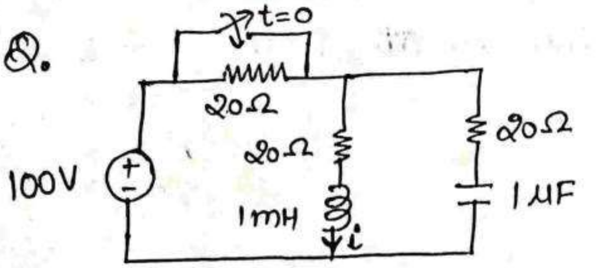
$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_c(0^+)}{C}$

$= \frac{1}{RC}$.

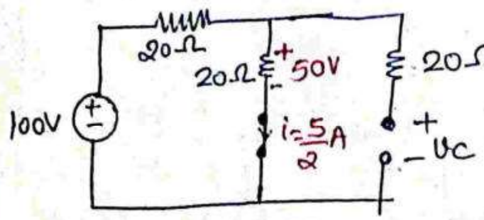
$i = C\frac{dv}{dt} \Rightarrow \frac{dv}{dt} = ?$

$v = L\frac{di}{dt} \Rightarrow \frac{di}{dt} = ?$

Q. find $\frac{di}{dt}$ at $t=0^+$



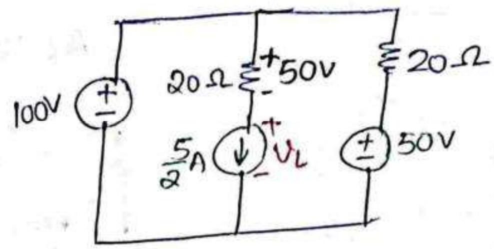
Sol^n At $t=0^-$



$i(0^-) = i(0^+) = \frac{5}{2}A$

$v_c(0^-) = v_c(0^+) = 50\ V$

At $t=0^+$



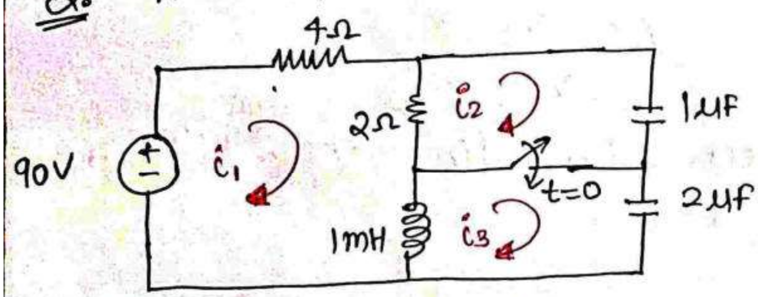
$V_L = L\frac{di}{dt}$

$\frac{di}{dt}\Big|_{t=0^+} = \frac{V(0^+)}{L} = \frac{50}{1m}$

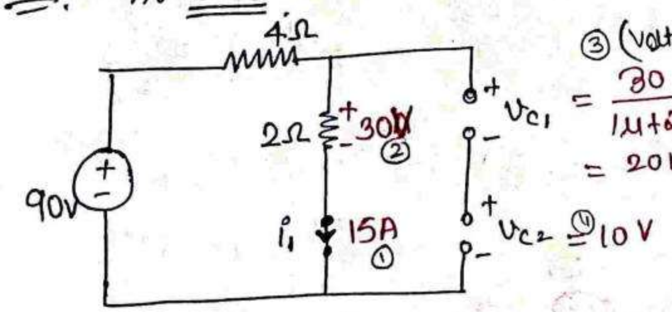
$= 50\ K\ A/s$

$V_L(0^+) = 50V$

Q. find i_1, i_2 & i_3 at $t=0$.



Sol^n At $\underline{t=0^-}$



③ (Voltage Division)

$v_{c1} = \frac{30}{1\mu + 2\mu} * 2\mu$

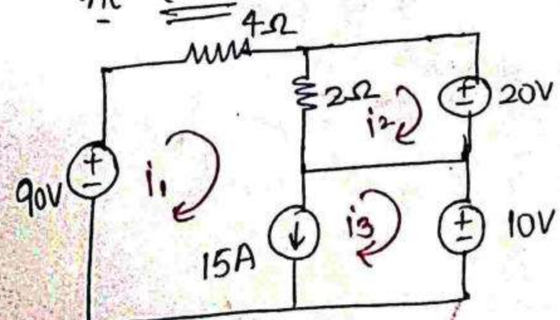
$= 20V$

$i_L(0^-) = i_L(0^+) = 15\ A$

$V_{c1}(0^-) = V_{c1}(0^+) = 20V$

$V_{c2}(0^-) = V_{c2}(0^+) = 10V$

At $\underline{t=0^+}$



$-90 + 4i_1 + 20 + 10 = 0$

- $i_1(0^+) = 15A$

KVL $20 + 2(i_2 - i_1) = 0$

$20 + 2i_2 - 2(15) = 0$

- $i_2(0^+) = 5A$

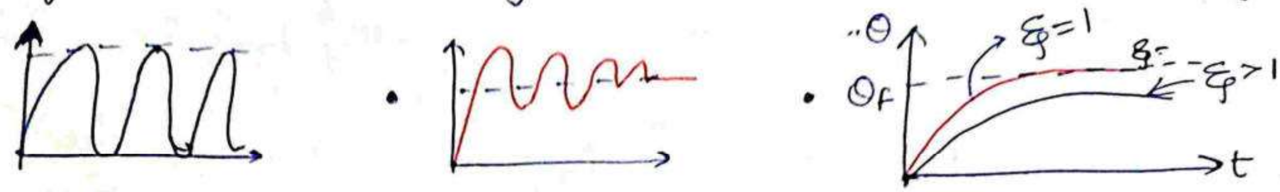
$i_1 - i_3 = 15$

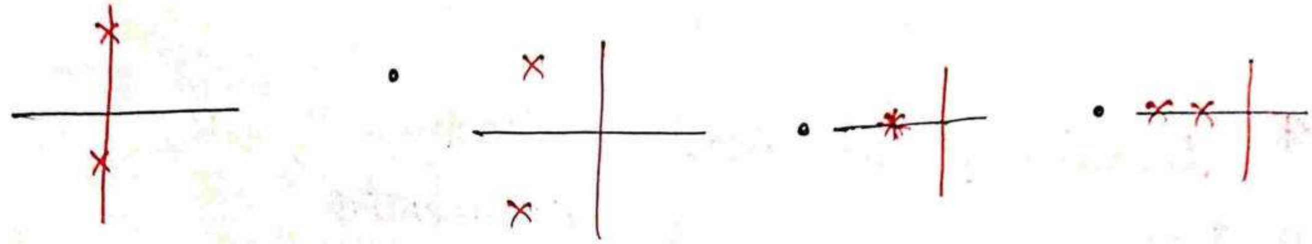
$i_3 = 15 - 15$

- $i_3(0^+) = 0\ A$

UNDAMPED | UNDERDAMPED | CRITICALLY | OVERDAMPED

- $\xi = 0$ | - $0 < \xi < 1$ | - $\xi = 1$ | - $\xi > 1$





$T_d = GI$

$T_c = K\theta$

i) AT $\theta = \theta_f$

$T_c = T_d$

$K\theta_f = GI$

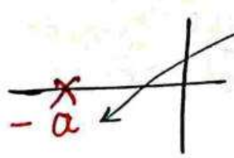
ii) $\theta > \theta_f$

$T_c > T_d$

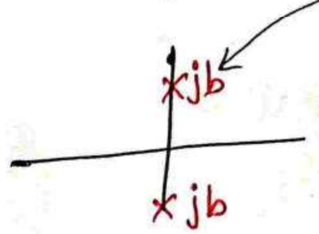
iii) $\theta < \theta_f$

$T_c < T_d$.

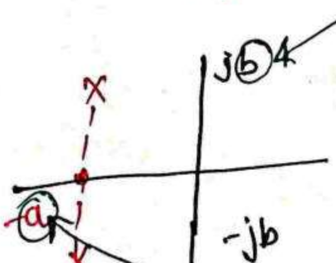
Overdamping ratio (ξ) = $\dfrac{\text{actual damping}}{\text{critical damping}}$



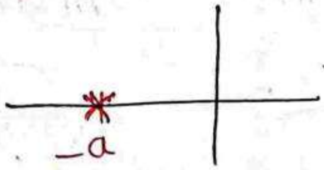
$\frac{1}{s+a} \longleftrightarrow e^{-at}$



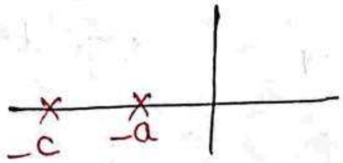
$\frac{1}{b}\,\frac{b}{s^2+b^2} \longleftrightarrow \frac{1}{b}\sin bt$



$\frac{1}{b}\cdot\frac{b}{(s+a)^2+b^2} \longleftrightarrow e^{-at}\sin bt$



$$\frac{1}{(S+a)^2} \Leftrightarrow t.e^{-at}$$



$$\frac{1}{(S+a)(S+c)} \Leftrightarrow \frac{K_1}{(S+a)} + \frac{K_2}{S+c}$$

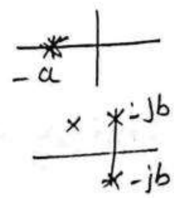
$$\Updownarrow$$

$$K_1 e^{-at} + K_2 e^{-ct}$$

- Time constant $(\tau) = (-\overset{ex.}{2} + j3)$

  $\hookrightarrow \tau = \frac{1}{2}$

* Location of the Poles	Nature of I.R (Impulse Response)
1) Real	Exponential
2) Imaginary	Sinusoidal
3) Real + Imaginary	Exponential $\times$ Sinusoidal.
4) Repeated poles of order 'n'	$t^{n-1} \times$ I.R of Simple pole.

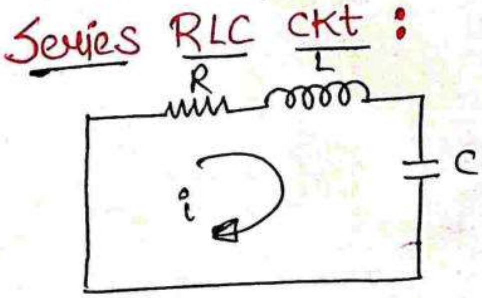


$t^{2-1}(e^{-at})$

$t.e^{-at}$

$\rightarrow \frac{1}{b} t \sin bt$

Series RLC Ckt :



$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

$$S^2 + \frac{R}{L} S + \frac{1}{LC} = 0$$

$$S_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$S_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Case-1 : $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

Overdamped $(\xi > 1)$

$$\alpha = \frac{-R}{2L}, \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$i(t) = C_1 e^{(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}$$

Case-2: $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

$$S_{1,2} = \frac{-R}{2L} = \alpha$$

critically damped. $(\xi = 1)$

$$i(t) = (C_1 + C_2 t)\, e^{\alpha t}$$

Case-4: $R = 0$

$$S_{1,2} = \pm j\frac{1}{\sqrt{LC}} = \beta$$

Undamped $(\xi = 0)$

$$i(t) = c_1 \cos\beta t + c_2 \sin\beta t$$

Case-3: $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$$S_{1,2} = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

under damped $(0 < \xi < 1)$

$$\alpha = \frac{-R}{2L} \quad \beta = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$i(t) = (c_1 \cos\beta t + c_2 \sin\beta t)\, e^{\alpha t}$$

$$\tau = \frac{1}{DC} = \frac{1}{\alpha}$$

$$\boxed{\tau = \frac{2L}{R}}$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

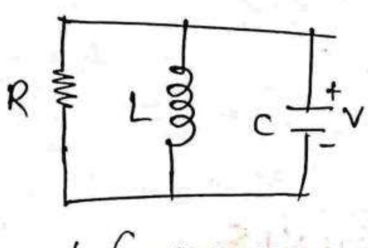
$$= \frac{1}{\sqrt{LC}}\sqrt{1 - \left(\frac{R}{2}\sqrt{\left(\frac{C}{L}\right)}\right)^2}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

Parallel RLC Ckt –



$$\frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int V\,dt = 0$$

$$C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{V}{L} = 0$$

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V}{LC} = 0$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$S_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Case-1 : $\left(\frac{1}{2RC}\right)^2 > \frac{1}{LC}$

Overdamped ($\xi > 1$)

$\alpha = \frac{-1}{2RC}$, $\beta = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$

$v(t) = c_1 e^{(\alpha+\beta)t} + c_2 e^{(\alpha-\beta)t}$

Case-2 : $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC}$

$S_{1,2} = \frac{-1}{2RC} = \alpha$

Critically damped ($\xi = 1$)

$v(t) = (c_1 + c_2 t) e^{\alpha t}$

Case-3 : $\left(\frac{1}{2RC}\right)^2 < \frac{1}{LC}$

$S_{1,2} = \frac{-1}{2RC} \pm j\sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$

Under damped ($0 < \xi < 1$)

$\alpha = \frac{-1}{2RC}$, $\beta = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$

$v(t) = (c_1 \cos\beta t + c_2 \sin\beta t) e^{\alpha t}$

Case-4 :

$\xi = 0$ ($R = \infty$)

$S_{1,2} = \pm j \frac{1}{\sqrt{LC}}$

Undamped ($\xi = 0$)

$v(t) = c_1 \cos\beta t + c_2 \cos\beta t$.

$\tau = \frac{1}{DC}$

* $\boxed{\tau = 2RC}$

$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$

$= \frac{1}{\sqrt{LC}} \sqrt{1 - \left(\frac{1}{2R}\sqrt{\frac{L}{C}}\right)^2}$

$\omega_d = \omega_n \sqrt{1 - \xi^2}$

* $\boxed{\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}}$

$L, R \overset{=0}{,} C \rightarrow$ Undamped

Q.4 critically damped.

$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

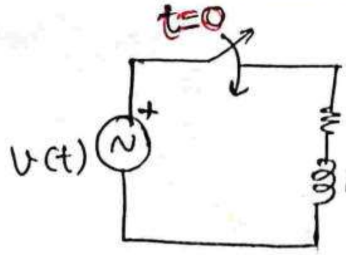
$\tau = \frac{2L}{R} = 0.5$ sec.

(b).

10 - d. , 16 - d

AC TRANSIENTS

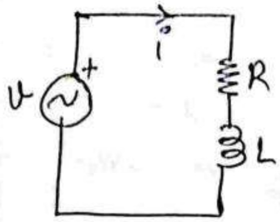
- DC transients are more severe as compared to the AC transients.
- In an AC ckt based on the selection of the ckt parameters, operating frequency & the switching operation. , it is possible to obtain the transient free respon.



$$v(t) = V_m \overset{\cos}{\sin}(\omega t+\theta)$$

$$i(0^-) = i(0^+) = 0$$

for $t>0$.



$$v = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{v}{L}$$

$$i(t) = i_{CF} + i_{PI}$$

i) CF :-

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$i_{CF} = Ae^{-\frac{R}{L}t}$$

ii) i_{PI} :-

$$i(\infty) = \frac{v(t)}{Z}$$

$$Z = R + j\omega L$$

$$|Z| = \sqrt{R^2+\omega^2L^2}$$

$$\alpha = \tan^{-1}\frac{\omega L}{R}$$

$$i_{PI} = \frac{V_m \overset{\cos}{\sin}(\omega t+\theta)}{|Z|\angle\alpha}$$

$$i_{PI} = \frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t+\theta-\alpha)$$

$$i(t) = Ae^{-\frac{R}{L}t} + \frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t+\theta-\alpha)$$

At $t=0$, $i=0$

$$0 = A\cdot 1 + \frac{V_m}{|Z|}\overset{\cos}{\sin}(\theta-\alpha)$$

$$i(t) = \underbrace{-\frac{V_m}{|Z|}\overset{\cos}{\sin}(\theta-\alpha)e^{-\frac{R}{L}t}}_{tr} + \frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t+\theta-\alpha)$$

$$\theta-\alpha = 0$$

$$\theta = \alpha$$

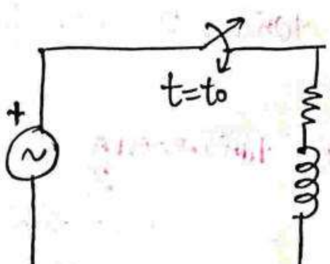
$$\boxed{\theta = \tan^{-1}\frac{\omega L}{R}}$$

$$\theta-\alpha = \frac{\pi}{2}$$

$$\theta = \alpha + \frac{\pi}{2}$$

$$\theta = \tan^{-1}\frac{\omega L}{R} + \frac{\pi}{2}$$

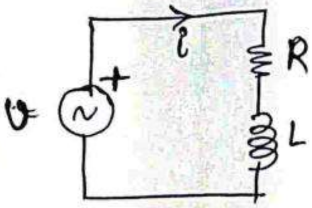
#



$$v(t) = V_m \sin(\omega t+\theta)$$

$$i(t_0^-) = i(t_0^+) = 0$$

for $t > t_0$.



$$v = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{v}{L}$$

$$i(t) = i_{CF} + i_{PI}$$

i) CF :

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$i_{CF} = Ae^{-\frac{R}{L}(t-t_0)}$$

ii) i_{PI} :

$$i(\infty) = \frac{v(t)}{Z}$$

$$Z = R + j\omega L$$

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

$$\alpha = \tan^{-1}\frac{\omega L}{R}$$

$$i_{PI} = \frac{V_m \overset{\cos}{\sin}(\omega t + \theta)}{|Z|\angle\alpha}$$

$$i_{PI} = \frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t + \theta - \alpha)$$

$$i(t) = Ae^{-\frac{R}{L}(t-t_0)} + \frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t + \theta - \alpha)$$

At $t = t_0$, $i = 0$

$$0 = A\cdot 1 + \frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t_0 + \theta - \alpha)$$

$$i(t) = \underbrace{-\frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t_0 + \theta - \alpha)e^{-\frac{R}{L}(t-t_0)}}_{tr} + \frac{V_m}{|Z|}\overset{\cos}{\sin}(\omega t + \theta - \alpha)$$

$$\omega t_0 + \theta - \alpha = 0$$

$$\omega t_0 = \alpha - \theta$$

$$\omega t_0 = \tan^{-1}\frac{\omega L}{R} - \theta$$

$$\omega t_0 + \theta - \alpha = \frac{\pi}{2}$$

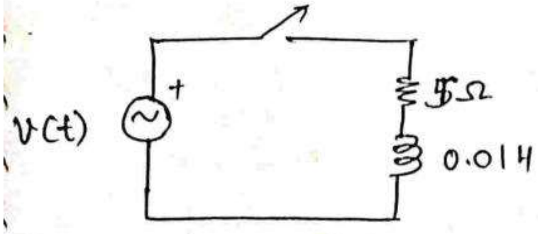
$$\omega t_0 = \alpha - \theta + \frac{\pi}{2}$$

$$\omega t_0 = \tan^{-1}\frac{\omega L}{R} - \theta + \frac{\pi}{2}$$

★★★

	RL		RC
1) $v(t) = V_m\sin(\omega t + \theta)$, $t = 0$	$\theta = \tan^{-1}\frac{\omega L}{R}$	$\theta = \tan^{-1}\omega\tau$	$\theta = \tan^{-1}\omega RC$
2) $v(t) = V_m\cos(\omega t + \theta)$, $t = 0$	$\theta = \tan^{-1}\frac{\omega L}{R} + \frac{\pi}{2}$	$\theta = \tan^{-1}\omega\tau + \frac{\pi}{2}$	$\theta = \tan^{-1}\omega RC + \frac{\pi}{2}$
3) $v(t) = V_m\sin(\omega t + \theta)$, $t = t_0$	$\omega t_0 = \tan^{-1}\frac{\omega L}{R} - \theta$	$\omega t_0 = \tan^{-1}\omega\tau - \theta$	$\omega t_0 = \tan^{-1}\omega RC - \theta$
4) $v(t) = V_m\cos(\omega t + \theta)$, $t = t_0$	$\omega t_0 = \tan^{-1}\frac{\omega L}{R} - \theta + \frac{\pi}{2}$	$\omega t_0 = \tan^{-1}\omega\tau - \theta + \frac{\pi}{2}$	$\omega t_0 = \tan^{-1}\omega RC - \theta + \frac{\pi}{2}$

Q. At what time $t = t_0$, switch must be operated so that the ckt current is free from transient.



$v(t) = V_m \sin \omega t$

$f = 50\,Hz.$

Sol^n $\omega t_0 = \tan^{-1} \frac{\omega L}{R}$ → radians

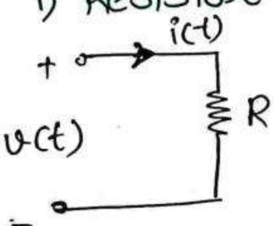
$$\omega t_0 = \tan^{-1} \frac{2\pi \times 50 \times 0.01}{5}$$

$$\omega t_0 = 0.561 \text{ rad.}$$

$$t_0 = \frac{0.561}{2 \times \pi \times 50} \quad \frac{\text{rad}}{\text{rad/sec}}, \quad t_0 = 1.786\,ms$$

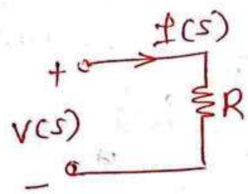
Application of Laplace Transform in transients :

1) Resistor

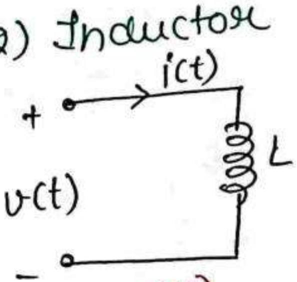


$v(t) = R\,i(t)$

$V(s) = R\,I(s)$



2) Inductor



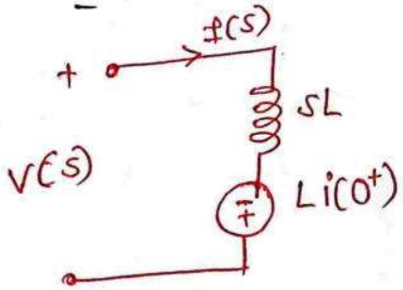
$$v(t) = L\frac{di}{dt}$$

$$V(s) = L\left[sI(s) - i(0^+)\right]$$

$$V(s) = sLI(s) - Li(0^+)$$

$$Z(s) = \frac{V(s)}{I(s)}$$

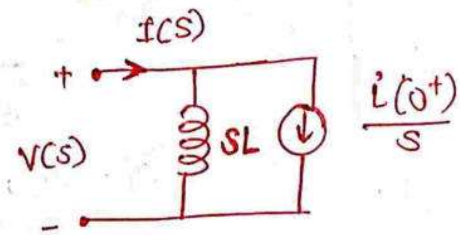
$$Z(s) = sL - \frac{Li(0^+)}{I(s)}$$



for current

$$sLI(s) = V(s) + Li(0^+)$$

$$I(s) = \frac{V(s)}{sL} + \frac{i(0^+)}{s}$$

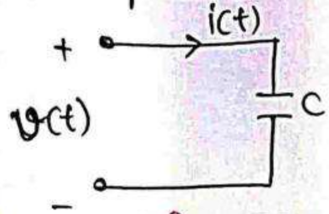


for admittance
↓

$$Y(s) = \frac{I(s)}{V(s)}$$

$$Y(s) = \frac{1}{sL} + \frac{i(0^+)}{sV(s)}$$

3) Capacitor



$$i = C\frac{dv}{dt}$$

$$Z(s) = \frac{V(s)}{I(s)}$$

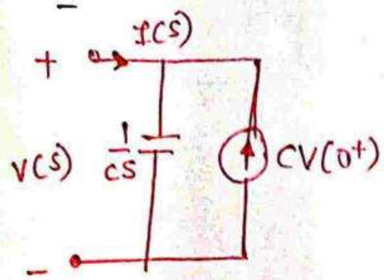
$$Z(s) = \frac{1}{Cs} + \frac{V(0^+)}{sI(s)}$$

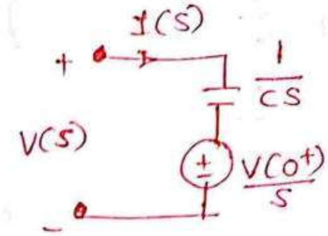
$$I(s) = C\left[sV(s) - V(0^+)\right]$$

$$I(s) = CsV(s) - CV(0^+)$$

$$I(s) = \frac{V(s)}{1/Cs} - CV(0^+)$$

$$Y(s) = \frac{I(s)}{V(s)}$$

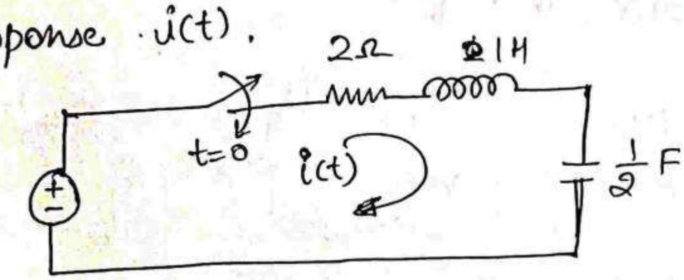




$$CsV(s) = I(s) + CV(0^+)$$

$$V(s) = \frac{1}{Cs}\cdot I(s) + \frac{V(0^+)}{s}$$

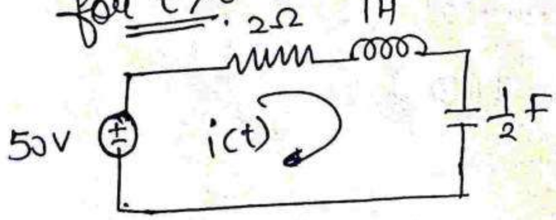
Q. In the below given electrical n/w, find the current response $i(t)$.



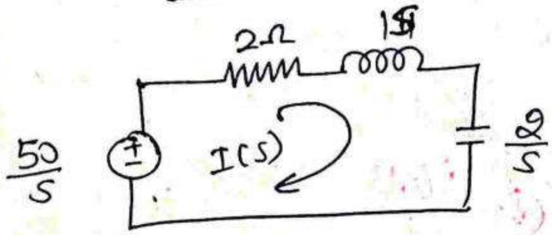
Sol^n $i(0^-) = i(0^+) = 0$

$V_c(0^-) = V_c(0^+) = 0$

for $t > 0$



T.V → L.D



$$I(s) = \frac{\frac{50}{s}}{2 + s + \frac{2}{s}}$$

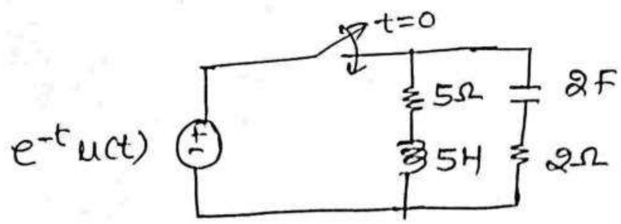
$$I(s) = \frac{50}{s^2+2s+2}$$

$$= \frac{50}{(s+1)^2 + 1^2}$$

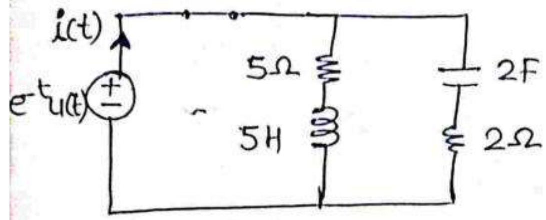
$$I(s) = 50\,\frac{1}{(s+1)^2+1^2}$$

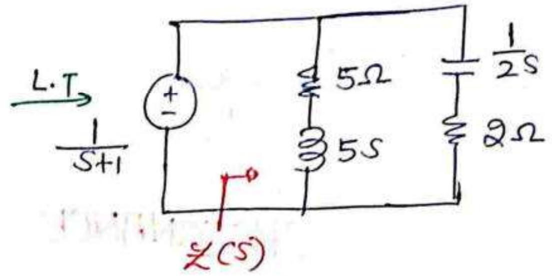
$$i(t) = 50e^{-t}\sin t \ A$$

Q. find the current response $i(t)$, for $t>0$.



for $t>0$.





$$Z(s) = \frac{(5+5s)(2+\frac{1}{2s})}{5+5s+2+\frac{1}{2s}}$$

$$Z(s) = \frac{5(s+1)(4s+1)}{10s^2+14s+1}$$

$$I(s) = \frac{V(s)}{Z(s)}$$

$$I(s) = \frac{10s^2+14s+1}{5(s+1)^2(4s+1)}$$

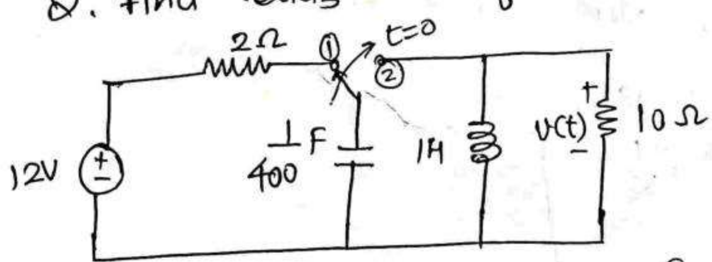
$$I(s) = \frac{a_1}{(s+1)^2} + \frac{a_1}{s+1} + \frac{a_2}{4s+1}$$

$$= \frac{1}{5(s+1)^2} + \frac{2}{3(s+1)} - \frac{2}{3(4s+1)}$$

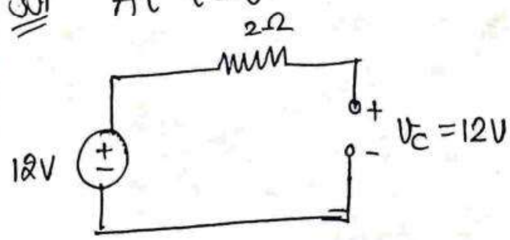
$$= \frac{1}{5(s+1)^2} + \frac{2}{3(s+1)} - \frac{1}{6(s+\frac{1}{4})}$$

$$i(t) = \frac{1}{5}te^{-t} + \frac{2}{3}e^{-t} - \frac{1}{6}e^{-\frac{1}{4}t} \text{ A.}$$

Q. find voltage $v(t)$ for $t>0$.

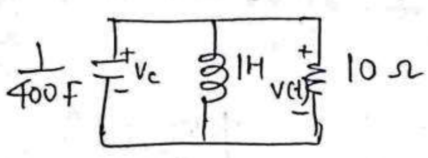


Sol^n At $t=0^-$

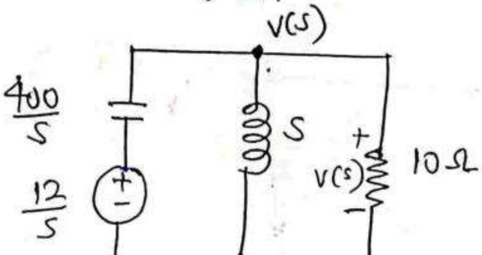


$V_C(0^-) = V_C(0^+) = 12$

for $t>0$.



↓ L.T



$$\frac{V(s) - \frac{12}{s}}{\frac{400}{s}} + \frac{V(s)}{s} + \frac{V(s)}{10} = 0$$

$$V_s\left[\frac{s}{400} + \frac{1}{s} + \frac{1}{10}\right] - \frac{12}{400} = 0$$

$$V_s\left[\frac{s^2 + 400 + 40s}{400s}\right] = \frac{12}{400}$$

$$V(s) = \frac{12s}{s^2 + 40s + 400}$$

$$V(s) = \frac{12s}{(s+20)^2}$$

$$= \frac{12(s+20) - 12 \times 20}{(s+20)^2}$$

$$= \frac{12}{s+20} - \frac{240}{(s+20)^2}$$

$$v(t) = 12e^{-20t} - 240te^{-20t}\,V.$$

SINUSOIDAL STEADY STATE ANALYSIS

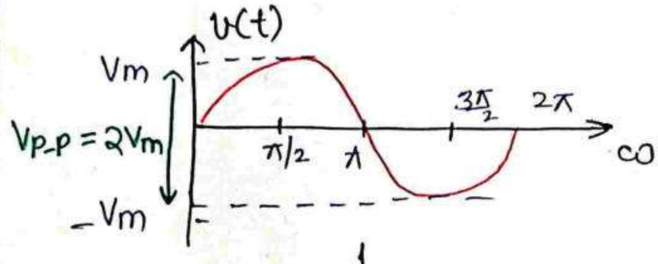
Sinusoidal
- sine
- Cosine

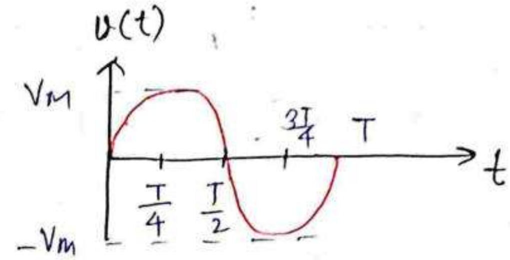
$v(t) = V_m \sin \omega t$

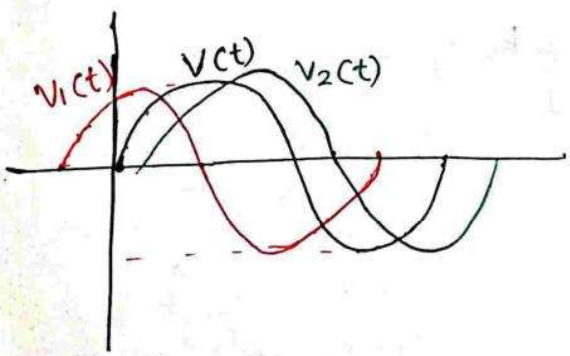
$V_m \rightarrow$ max. amplitude

$\omega \rightarrow$ Angular frequency (rad/sec)

$\omega t \rightarrow$ Argument (rad).







$v(t) = V_m \sin \omega t$

$v_1(t) = V_m \sin(\omega t + \theta)$

$v_2(t) = V_m \sin(\omega t - \theta)$

We can compare phases of 2 sinusoids if

1) If both the sinusoids have the same frequency.

2) Both the sinusoids are expressed in the same form i.e. either in sine or in cosine.

3) Both the sinusoids have +ve amplitude. (i.e. same sign w.r.t each other).

$v(t) = V_m \sin(\omega t + \theta)$

$e^{j\theta} = \cos\theta + j\sin\theta$

$= 1\angle\theta$

$Re(e^{j\theta}) = \cos\theta$

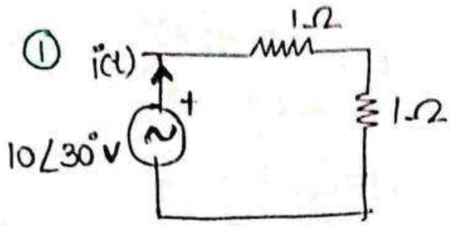
$Im(e^{j\theta}) = \sin\theta$

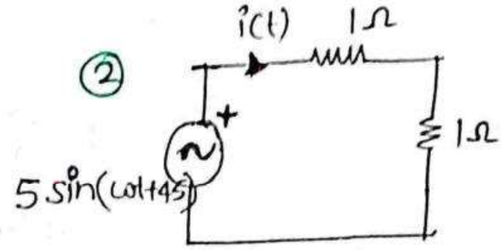
$$v(t) = \mathrm{Re}\{V_m e^{j(\omega t+\theta)}\}$$

$$v(t) = \mathrm{Re}\{V_m e^{j\theta} \cdot e^{j\omega t}\}$$

$$\bar{V} = V_m e^{j\theta}$$

$$\bar{V} = V_m \angle\theta$$

① 

② 

$$\bar{I} = \frac{10\angle 30^\circ}{2}$$

$$\bar{I} = 5\angle 30^\circ \text{ A}$$

$$i(t) = \underbrace{5\sqrt{2}}_{\text{peak}} \cos(\omega t + 30^\circ)$$

$$\bar{V} = 5\angle 45^\circ \text{ V}$$

$$\bar{I} = \frac{5}{2}\angle 45^\circ \text{ A}$$

$$i(t) = \frac{5}{2}\sin(\omega t + 45^\circ) \text{ A}$$

- $$\bar{V}_{rms} = \frac{5}{\sqrt{2}}\angle 45^\circ \text{ V}$$

$$\bar{I}_{rms} = \frac{5}{2\sqrt{2}}\angle 45^\circ \text{ A}$$

$$i(t) = \frac{5}{2}\sin(\omega t + 45^\circ) \text{ A}.$$

- By supressing the time factor we transformed the sinusoid from the time domain to the phasor domain. thus the phasor transform transfers the sinusoidal functn from the time domain to the complex no. domain.

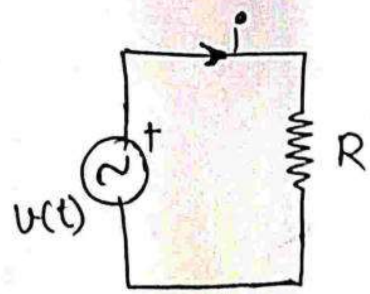
- Differences b/w $v(t)$ & $\bar{V}$.

1) $v(t)$ is the instantaneous or time domain respresentation while $\bar{V}$ is the phasor domain representation

2) $v(t)$ is time dependent. While $\bar{V}$ is time independent.

3) $v(t)$ is always real with no complex term. While $\bar{V}$ is generally complex.

① RESISTOR



$v(t) = V_m \sin\omega t$

$i(t) = \dfrac{v(t)}{R}$

$= \dfrac{V_m}{R} \sin\omega t$

$i(t) = I_m \sin\omega t$

- $P_{av} = \dfrac{1}{T}\int_0^T p(t)\,dt$

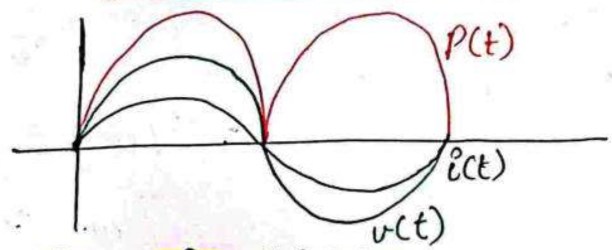
$= \dfrac{1}{T}\int_0^T v(t) . i(t) . dt$

$= \dfrac{1}{T}\int_0^T V_m \sin\omega t \cdot I_m \sin\omega t \, dt$

$= \dfrac{V_m I_m}{2T}\int_0^T (1 - \cos 2\omega t)\,dt$

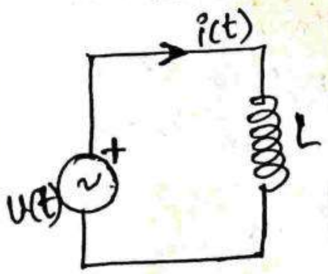
$P_{av} = \dfrac{V_m I_m}{2} = \dfrac{V_m}{\sqrt{2}} \cdot \dfrac{I_m}{\sqrt{2}}$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms}}$$



* $f_p = 2f_v$ (or) $2f_i$

② INDUCTOR



$i(t) = I_m \sin\omega t$

$v(t) = L\dfrac{di}{dt}$

$v(t) = L\dfrac{d}{dt}(I_m \sin\omega t)$

$v(t) = \omega L I_m \cos\omega t$

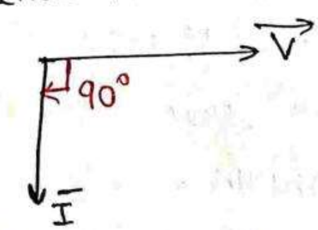
$v(t) = \omega L I_m \sin(\omega t + 90^\circ)$

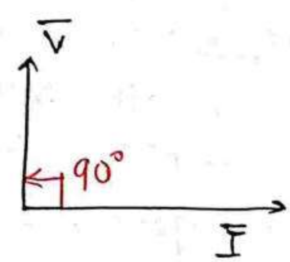
$\bar{V} = j\omega L \bar{I}$

$\bar{V} = jX_L \bar{I}$

Inductor
LIL ← lags
current

$X_L = \omega L$
↓
Inductive Reactance.





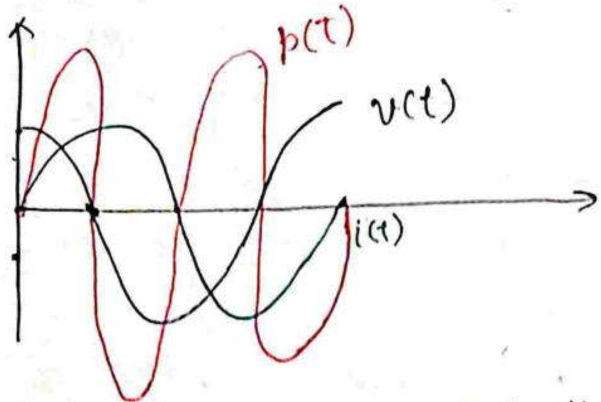
- $P_{av} = \dfrac{1}{T}\int_0^T p(t)\,dt$

$= \dfrac{1}{T}\int_0^T v(t) . i(t)\,dt$

$= \dfrac{1}{T}\int_0^T V_m \cos\omega t \cdot I_m \sin\omega t . dt$

$= \dfrac{V_m I_m}{2T}\int_0^T \sin 2\omega t\, dt$

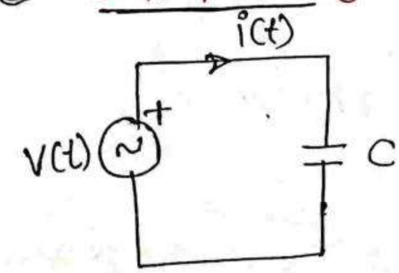
$$\boxed{P_{av} = 0}$$



$$* \; f_p = 2f_v \text{ (or) } 2f_i$$

- In the +ve half cycle of the power, Inductor takes the energy from the source & in the -ve half cycle of the power, Inductor delivers the energy to the source.

③ CAPACITOR



$$v(t) = V_m \sin \omega t$$

$$i = C \frac{dv}{dt}$$

$$i = C \frac{d}{dt} (V_m \sin \omega t)$$

$$i = \omega C V_m \cos \omega t$$

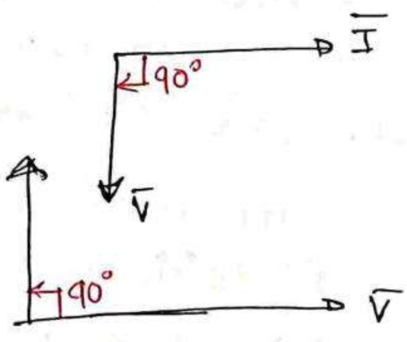
$$i = \omega C V_m \sin(\omega t + 90^\circ)$$

$$\bar{I} = j\omega C \bar{V}$$

$$\bar{V} = \frac{1}{j\omega C} \bar{I}$$

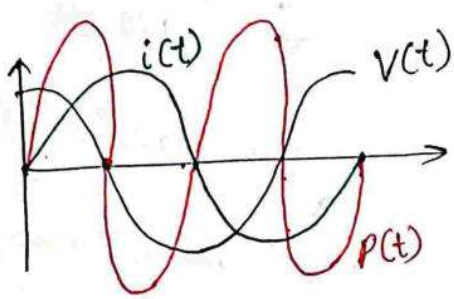
$$\bar{V} = -j\frac{1}{\omega C} \bar{I}$$

$$\bar{V} = -jX_C \bar{I}$$



$$X_C = \frac{1}{\omega C}$$

Capacitive Reactance



$$\cdot \; P_{av} = \frac{1}{T} \int_0^T P(t)\, dt$$

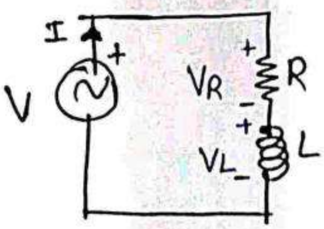
$$= \frac{1}{T} \int_0^T v(t) \cdot i(t)\, dt$$

$$= \frac{1}{T} \int_0^T V_m \sin \omega t \cdot I_m \cos \omega t \, dt$$

$$* \; f_p = 2f_v \text{ (or) } 2f_i$$

$$\boxed{P_{av} = 0}$$

① Series RL circuit

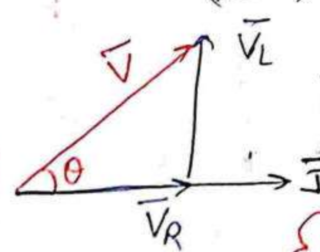


$$\bar{V} = \bar{V}_R + \bar{V}_L$$

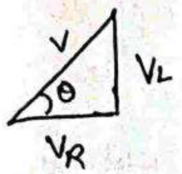
$$\bar{I}Z = \bar{I}R + jX_L\bar{I}$$

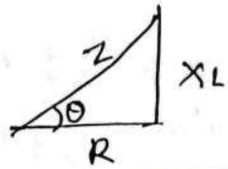
$$\boxed{Z = R + jX_L}$$

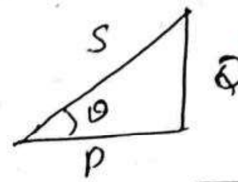
(Phasor)



$P = I^2R$
$Q = I^2X_L$
$S = I^2Z$
$pf = \frac{P}{S}$







$$V = \sqrt{V_R^2 + V_L^2}$$

$$\theta = \tan^{-1}\frac{V_L}{V_R}$$

$$\cos\theta = \frac{V_R}{V} \text{ (lagging)}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1}\frac{X_L}{R}$$

$$\cos\theta = \frac{R}{Z} \text{ (lagging)}$$

$$S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1}\frac{Q_L}{P}$$

$$\cos\theta = \frac{P}{S} \text{ (lagging)}$$

- Pf angle indicates the position of current phasor w.r.t voltage phasor.
- While defining the Pf for any circuit, voltage phasor is taken as reference bcz in the real time system all the loads are connected in parallel.
- Pf tells us that how much of the apparent power is utilized. If the Pf is high, then active power delivered is also high.

. [illegible]
. Phase angle 0 → R
, " 90° → add.
180° → Subtract

- $P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$

$$= \frac{1}{T}\int_0^T v(t).\,i(t)\,dt$$

$$= \frac{1}{T}\int_0^T V_m\sin(\omega t+\theta).\,I_m\sin\omega t\,dt$$

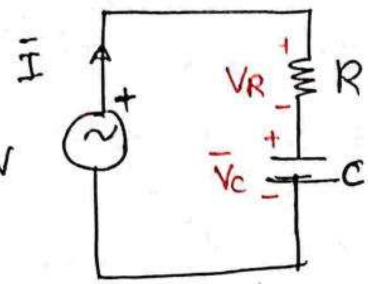
$$= \frac{V_mI_m}{2T}\int_0^T\{\cos\theta - \cos(2\omega t+\theta)\}\,dt$$

$$P_{av} = \frac{V_m \cdot I_m}{2} \cos\theta$$

$$P_{av} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\theta \cdot$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cdot \cos\theta}$$

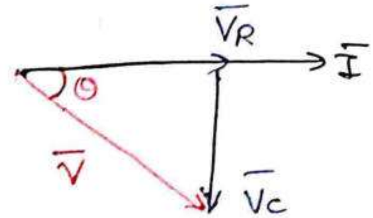
(2) Series RC circuit

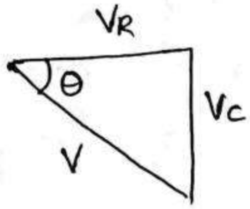


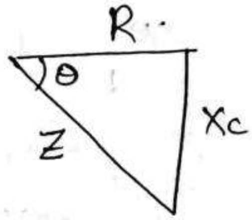
$$\bar{V} = \bar{V}_R + \bar{V}_C$$

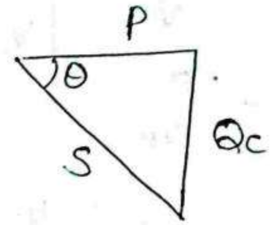
$$\bar{I}Z = IR - jX_C I$$

$$\boxed{Z = R - jX_C}$$









$$V = \sqrt{V_R^2 + V_C^2}$$

$$\theta = \tan^{-1}\left(\frac{-V_C}{V_R}\right)$$

$$\cos\theta = \frac{V_R}{V} \text{ (leading)}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$\cos\theta = \frac{R}{Z} \text{ (leading)}$$

$$S = \sqrt{P^2 + Q_C^2}$$

$$\theta = \tan^{-1}\left(\frac{-Q_C}{P}\right)$$

$$\cos\theta = \frac{P}{S} \text{ (leading)}$$

- $P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$

$$= \frac{1}{T}\int_0^T v(t) \cdot i(t)\,dt$$

$$= \frac{1}{T}\int_0^T I_m \sin(\omega t + \theta) \cdot V_m \sin\omega t \cdot dt$$

$$= \frac{V_m I_m}{2T}\int_0^T (\sin\omega t + \theta) \cdot \sin\omega t \cdot dt$$

$$P_{av} = \frac{V_m I_m}{2}\cos\theta = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\theta$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cdot \cos\theta}$$

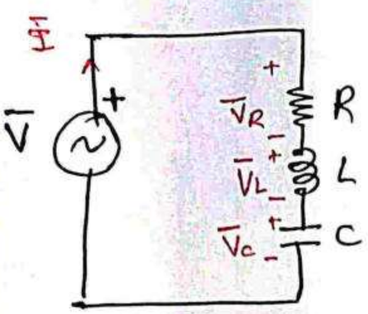
* Impedance ~~angle~~ & pf angle shows opposite variations

$$\bar{I} = \frac{\bar{V}}{\bar{Z}}$$

$$\bar{I} = \frac{\bar{V}}{|Z| \angle\theta}$$

$$\bar{I} = \frac{\bar{V}}{|Z|} \angle -\theta$$

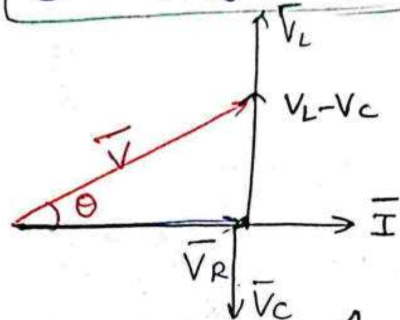
③ Series RLC circuit

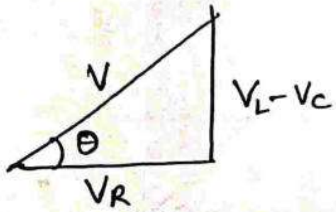


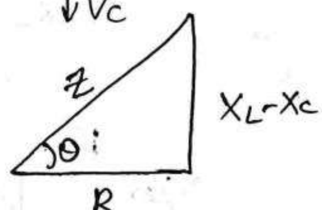
$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_c$$

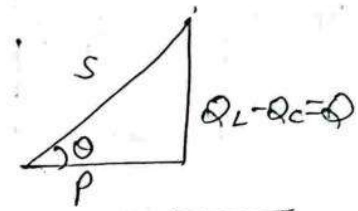
$$\bar{I}Z = \bar{I}R + jX_L\bar{I} - jX_C\bar{I}$$

$$\boxed{Z = R + jX_L - jX_C}$$









$$V = \sqrt{V_R^2 + (V_L - V_c)^2}$$

$$\theta = \tan^{-1}\frac{V_L - V_c}{V_R}$$

$$\cos\theta = \frac{V_R}{V}$$

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

$$\theta = \tan^{-1}\frac{X_L - X_c}{R}$$

$$\cos\theta = \frac{R}{Z}$$

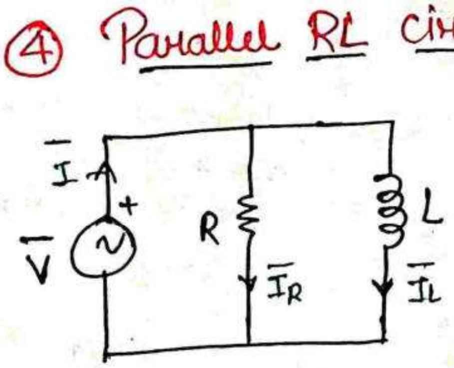
$$S = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1}\frac{Q}{P}$$

$$\cos\theta = \frac{P}{S}$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos\theta}$$

④ Parallel RL circuit



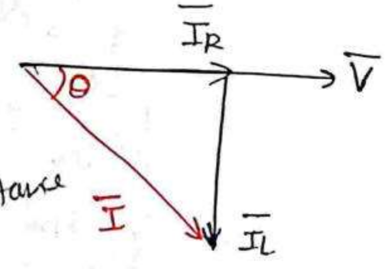
$$\bar{I} = \bar{I}_R + \bar{I}_L$$

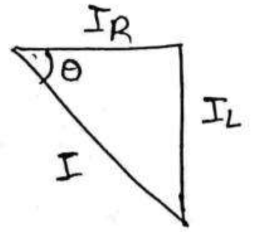
$$\frac{\bar{V}}{Z} = \frac{\bar{V}}{R} + \frac{\bar{V}}{jX_L}$$

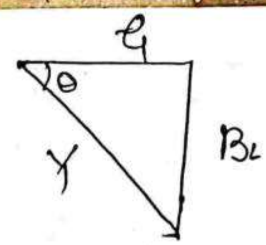
$$\frac{1}{Z} = \frac{1}{R} - j\left(\frac{1}{X_L}\right) \leftarrow \text{susceptance}$$

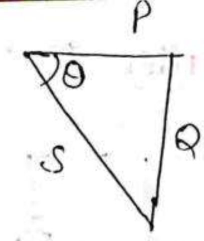
$$\boxed{Y = G - jB_L}$$

conductance









$I = \sqrt{I_R^2 + I_L^2}$

$\theta = \tan^{-1}\left(\frac{-I_L}{I_R}\right)$

$\cos\theta = \frac{I_R}{I}$ (lagging)

$Y = \sqrt{G^2 + B_L^2}$

$\theta = \tan^{-1}\left(\frac{-B_L}{G}\right)$

$\cos\theta = \frac{G}{Y}$ (lagging)

$S = \sqrt{P^2 + Q^2}$

$\theta = \tan^{-1}\left(\frac{-Q_L}{P}\right)$

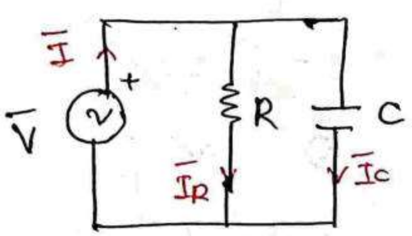
$\cos\theta = \frac{P}{S}$ (lagging)

- $\bar{I} = \bar{V} Y$

$\bar{I} = \bar{V} \, |Y| \angle\theta$

Impedance ~~angle~~ or Pf angle shows same variation.

⑤ Parallel RC circuit

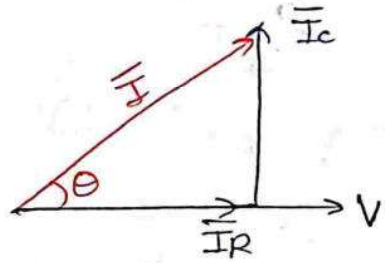


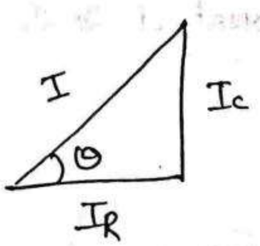
$\bar{I} = \bar{I_R} + \bar{I_C}$

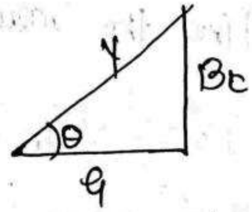
$\frac{\bar{V}}{Z} = \frac{\bar{V}}{R} + \frac{\bar{V}}{-jX_C}$

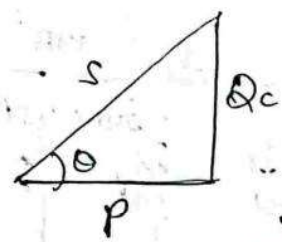
$\frac{1}{Z} = \frac{1}{R} + j\frac{1}{X_C}$

$$\boxed{Y = G + jB_C}$$









$I = \sqrt{I_R^2 + I_C^2}$

$\theta = \tan^{-1}\frac{I_C}{I_R}$

$\cos\theta = \frac{I_R}{I}$ (leading)

$Y = \sqrt{G^2 + B_C^2}$

$\theta = \tan^{-1}\frac{B_C}{G}$

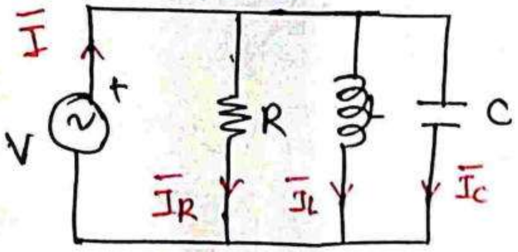
$\cos\theta = \frac{G}{Y}$ (leading)

$S = \sqrt{P^2 + Q_L^2}$

$\theta = \tan^{-1}\frac{Q_L}{P}$

$\cos\theta = \frac{P}{S}$ [leading]

⑥ Parallel RLC circuit –



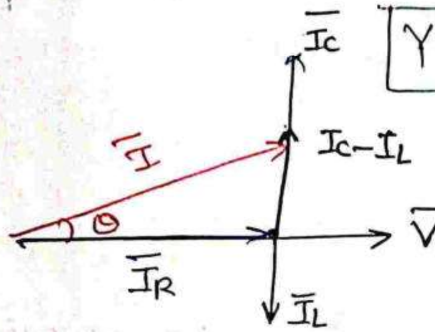
$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$\frac{\bar{V}}{Z} = \frac{\bar{V}}{R} + \frac{\bar{V}}{jX_L} + \frac{\bar{V}}{-jX_C}$$

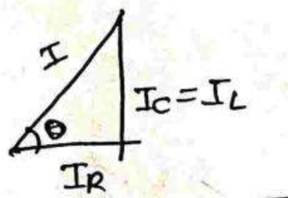
$$\frac{1}{Z} = \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

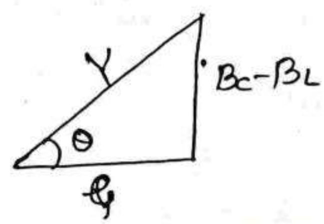
$$Y = G - jB_L + jB_C$$

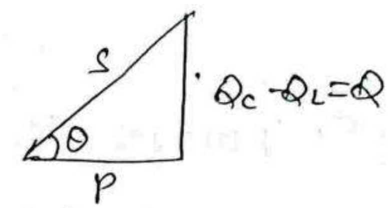
$$\boxed{Y = G + j(B_C - B_L)}$$



$|I_C| > |I_L|$







$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\theta = \tan^{-1}\frac{I_C - I_L}{I_R}$$

$$\cos\theta = \frac{I_R}{I}$$

$$Y = \sqrt{G^2 + (B_C - B_L)^2}$$

$$\theta = \tan^{-1}\frac{B_C - B_L}{G}$$

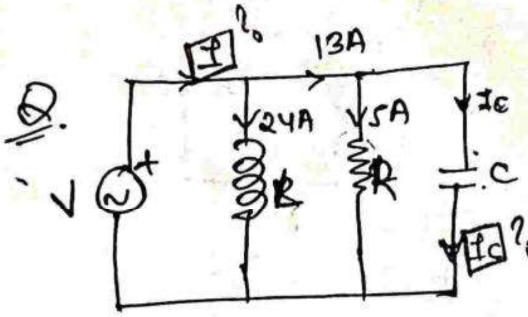
$$\cos\theta = \frac{G}{Y}$$

$$S = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1}\frac{Q}{P}$$

$$\cos\theta = \frac{P}{S}$$

Q.



Find the value of current. I & I_C

$$13 = \sqrt{I_R^2 + (I_C)^2}$$

$$(13)^2 = (5)^2 + I_C^2$$

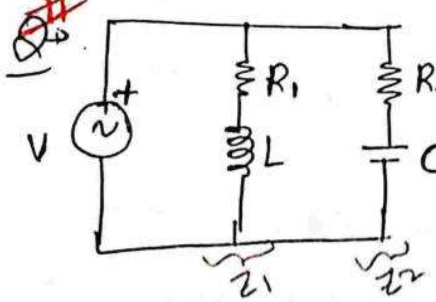
$$I_C = \sqrt{169 - 25}$$

$$\boxed{I_C = 12A}$$

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$= \sqrt{(5)^2 + (12-24)^2} = \sqrt{25 + 144} = 13A.$$

Q→



- $Z_1 = R_1 + jX_L$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L}$$

$$Y_1 = \frac{1}{R_1 + jX_L} \cdot \frac{R_1 - jX_L}{R_1 - jX_L}$$

$$Y_1 = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$$

$$Y_1 = G_1 - jB_L$$

$$\boxed{G_1 = \frac{R_1}{R_1^2 + X_L^2} \qquad B_L = \frac{X_L}{R_1^2 + X_L^2}}$$

- $Z_2 = R_2 - jX_C$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C}$$

$$Y_2 = \frac{1}{R_2 - jX_C} \cdot \frac{R_2 + jX_C}{R_2 + jX_C}$$

$$Y_2 = \frac{R_2}{R_2^2 + X_L^2} + j\frac{X_C}{R_2^2 + X_L^2}$$

$$Y_2 = G_2 + jB_C$$

$$\boxed{G_2 = \frac{R_2}{R_2^2 + X_L^2} \qquad B_C = \frac{X_C}{R_2^2 + X_C^2}}$$

$$Y = Y_1 + Y_2$$

$$Y = (G_1 - jB_L) + (G_2 + jB_C)$$

$$Y = (G_1 + G_2) + j(B_C - B_L)$$

$$I = VY$$

- $Z = x + jy$
$= re^{j\theta}$
$= r\angle\theta$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

- $Z_1 = x_1 + jy_1 = r_1\angle\theta_1$
- $Z_2 = x_2 + jy_2 = r_2\angle\theta_2$

① Addition

$$Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

② Substraction

$$Z_1 - Z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

③ Multiplication

$$Z_1 \cdot Z_2 = r_1 \cdot r_2 \angle \theta_1 + \theta_2$$

④ Division

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

⑤ Reciprocal.

$$\frac{1}{Z} = \frac{1}{r} \angle -\theta$$

⑥ Square root.

$$\sqrt{Z} = \sqrt{r} \angle \frac{\theta}{2}$$

⑦ Conjugate.

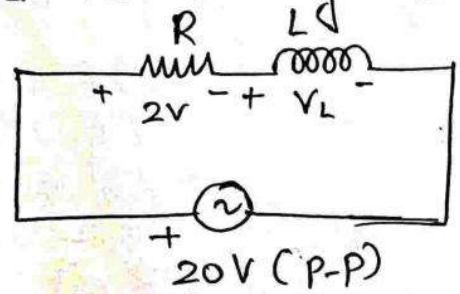
$$Z^* = x - jy = r \angle -\theta$$

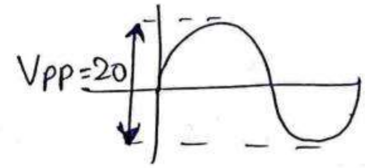
- $Z^n = (re^{j\theta})^n = r^n e^{jn\theta} = r^n \angle n\theta$
- $Z_1 Z_2 = (r_1 e^{j\theta_1})(r_2 e^{j\theta_2})$

$= r_1 r_2 e^{j(\theta_1+\theta_2)}$

$= r_1 r_2 \angle \theta_1 + \theta_2$

Q. Find the value of voltage V.





$V_{pp} = 20$

$20 = 2V_m$

$V_m = 10$ V.

Sol^n

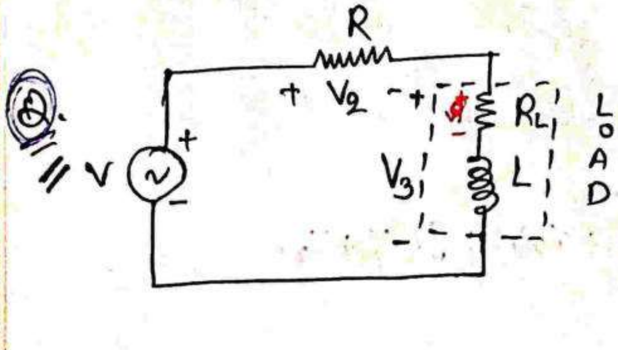
$$V_{RMS} = \sqrt{V_R^2 + V_L^2}$$

$$\frac{10}{\sqrt{2}} = \sqrt{(2)^2 + V_L^2}$$

$$50 = 4 + V_L^2$$

$$\boxed{V_L = \sqrt{46}\ V}$$

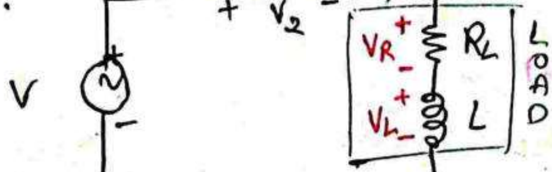
Q.

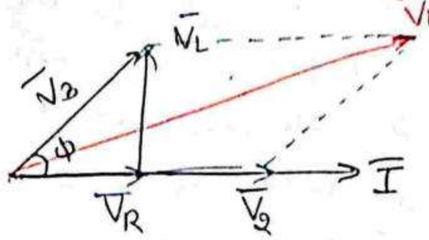


In the given electrical n/w find the pf of the load & power dissipation in the load when load resistance R_L is 5Ω.

$V_1 = 220V$, $V_2 = 122V$, $V_3 = 136V$.

Sol^n.





$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$220 = \sqrt{(136)^2 + (122)^2 + 2(122)(136)\cos\phi}$$

$$\boxed{\cos\phi = 0.45} \text{ lag.}$$

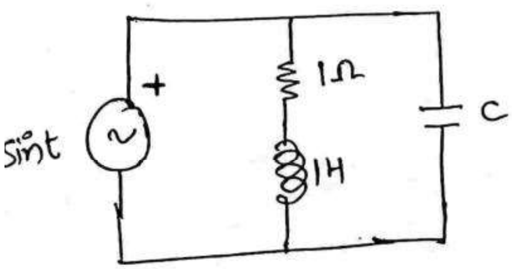
- $\cos\phi = \dfrac{V_R}{V_3}$

$$V_R = (0.45)(136)$$

$$V_R = 61.2\,V.$$

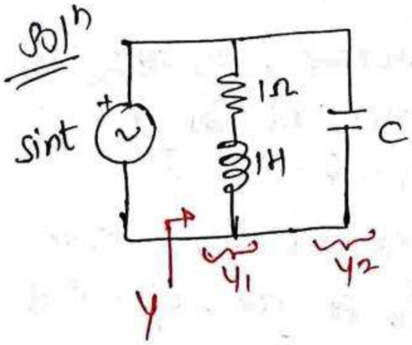
- $P = \dfrac{V_R^2}{R_L} = \dfrac{(61.2)^2}{5} = 749.1\ W.$

Q. Find the value of capacitance C when the Pf of the CKt is 0.8 lag.



- $X_L = \omega L$
$= (1)(1)$
$X_L = 1\Omega$

Sol^n



$$Y_1 = G_1 - jB_L$$

$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$$

$$= \frac{1}{1^2+1^2} - j\frac{1}{1^2+1^2}$$

$$Y_1 = \frac{1}{2} - j\frac{1}{2}$$

$$Y_2 = jB_C$$

$$= j\omega C$$

$$Y_2 = jC$$

$$Y = Y_1 + Y_2 = \frac{1}{2} + j\left(C - \frac{1}{2}\right)$$

$$\cos\theta = \frac{G}{Y}$$

$$0.8 = \frac{1/2}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(C - \frac{1}{2}\right)^2}}, \quad C = \frac{7}{8}, \frac{1}{8}$$

As Pf is lagging

$B_L > B_C$

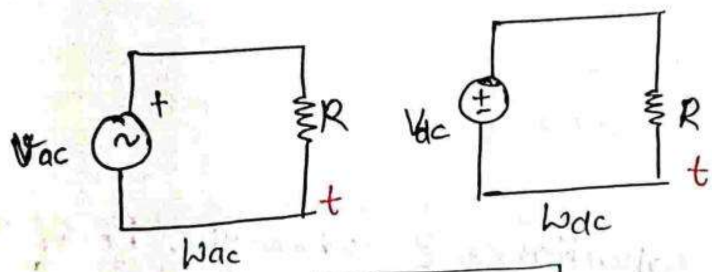
$\frac{1}{2} > C$

$C < \frac{1}{2}$

$\therefore C = \frac{1}{8}\ F$

Root mean square value : [RMS value]

- RMS value is defined based on the heating effect of the w/f.
- The voltage at which the heat dissipated in the ac ckt is equal to the heat dissipated in the dc ckt is called as the rms value, provided both ac & dc ckts have equal value of resistance & are operated for the same time interval.



$$\boxed{W_{ac} = W_{dc}}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 d\omega t}$$

Average value :

It is defined based on the charge transfer in the ckt.

The voltage at which the charge transfered in a ac ckt is equal to the charge transferred in a dc ckt is called as average value, Provided both ac & dc ckts have equal value of resistance & are operated for the same time interval.

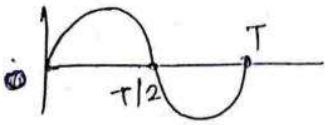
1) Unsymmetrical

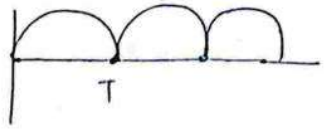
$$V_{av} = \frac{1}{T}\int_0^T v\,dt$$

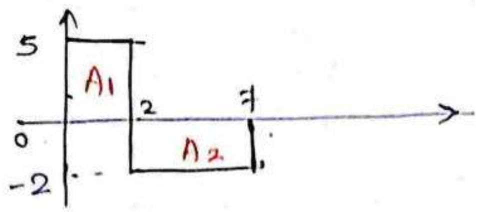
2) Symmetrical.

$V_{av} = 0 \longrightarrow$ full cycle.

$V_{av} = \frac{1}{T/2}\int_0^{T/2} v\,dt \longrightarrow$ Half cycle.







$A_1 = +10$

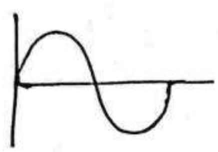
$A_2 = -10$ -Ve Area.

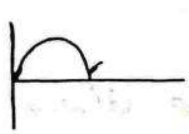
↓+Ve area ↓

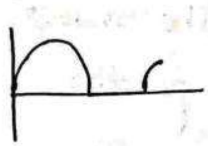
$|A_1| = |A_2| \Rightarrow$ Symmetry.

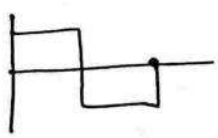
$|A_1| \neq |A_2| \Rightarrow$ Unsymmetrical.

* Vrms Vav

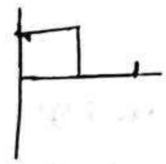
 $\frac{V_m}{\sqrt{2}}$ $\frac{2V_m}{\pi}(0)$

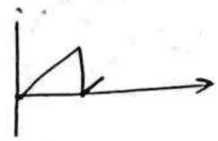
 $\frac{V_m}{\sqrt{2}}$ $\frac{2V_m}{\pi}$

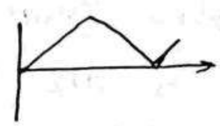
 $\frac{V_m}{2}$ $\frac{V_m}{\pi}$

 V_m $V_m(0)$

Vrms Vav

 $\frac{V_m}{\sqrt{2}}$ $\frac{V_m}{2}$

 $\frac{V_m}{\sqrt{3}}$ $\frac{V_m}{2}$

 $\frac{V_m}{\sqrt{3}}$ $\frac{V_m}{2}$

Form factor

It is a ratio of the ~~ratio~~ rms value of the w/f to the avg value of the w/f.

$$F.F = \frac{\text{RMS value}}{\text{Avg value}} = \frac{V_{rms}}{V_{av}} = \frac{I_{rms}}{I_{av}}$$

Peak factor.

It is a ratio of the max^n value of a w/f to the rms value of the w/f.

$$Pf = \frac{\text{Peak value}}{\text{RMS value}}$$

↳ $v(t) = V_0 + V_1 \sin \omega t + V_3 \sin 3\omega t$

- $V_{av} = V_0$
- $V_{rms} = \sqrt{V_{rms_1}^2 + V_{rms_2}^2 + \cdots}$

$$V_{rms} = \sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_m}{\sqrt{2}}\right)^2}$$

↳ $v(t) = V_0 + V_1(\sin \omega t + \theta_1) + V_2 \sin(\omega t - \theta_2)$

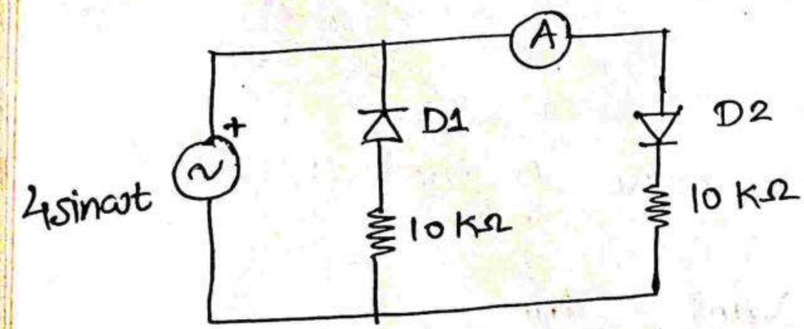
$= V_0 + V_1 \angle \theta_1 + V_2 \angle -\theta_2$

$= V_0 + V \angle \theta$

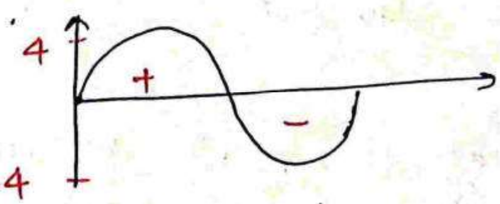
$= V_0 + V \sin(\omega t + \theta)$

$$V_{rms} = \sqrt{V_0^2 + \left(\frac{V}{\sqrt{2}}\right)^2}$$

Q. In the below given electrical n/ω, both the diodes D1 & D2 & the ammeter are ideal & the ammeter indicates the avg value. Find the reading of the ammeter.



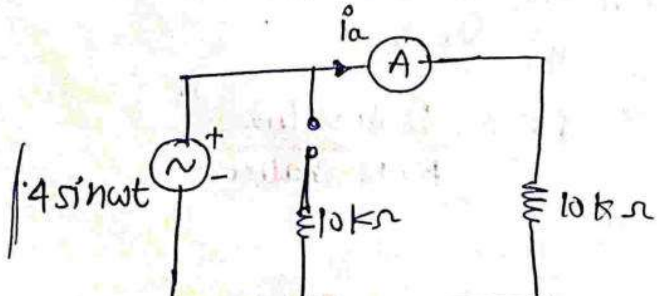
Sol^n.



1) +ve Half cycle —

D1 → RB → OC

D2 → FB → SC



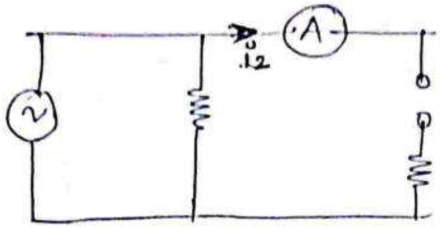
$$i_a(t) = \frac{4 \sin \omega t}{10K}$$

$$i_a(t) = 0.4 \sin \omega t \ \ mA.$$

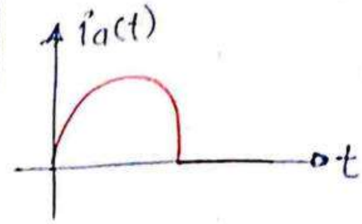
2) -ve half cycle -

D1 → FB → SC

D2 → RB → OC

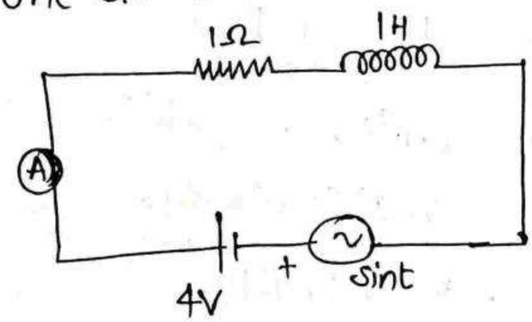


$i_{a}(t) = 0$

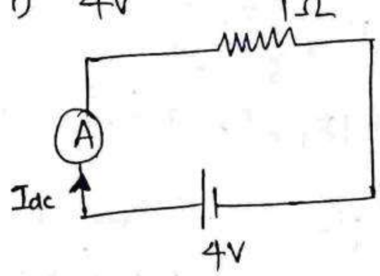


$$\text{Ammeter reading} = \frac{I_m}{\pi} = \frac{0.4}{\pi} \ mA.$$

Q. Find the reading of ammeter & Pf of the ckt shown in the fig. below.

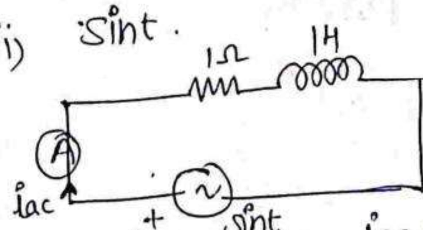


(a) i) 4V



$I_{dc} = 4A.$

ii) sint.



$$X_L = \omega L = (1)(1)$$

$$X_L = 1\Omega$$

$$i_{ac} = \frac{\sin t}{1+j1} = \frac{\sin t}{\sqrt{2} \angle 45^\circ}$$

$$i_{ac} = \frac{1}{\sqrt{2}} \sin(t - 45^\circ)$$

$$I_{ammeter} = \sqrt{I_{rms_1}^2 + I_{rms_2}^2} = \sqrt{4^2 + \left(\frac{1}{2}\right)^2} = \sqrt{16.25} \ A$$

(b) $\cos\theta = \frac{P}{S} = \frac{I_{rms}^2 R}{V_{rms} \cdot I_{rms}}$

$$\cos\theta = \frac{I_{rms} R}{V_{rms}}$$

$$V_{rms} = \sqrt{V_{rms_1}^2 + V_{rms_2}^2} = \sqrt{4^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{16.5}\ V$$

$$\cos\theta = \frac{(\sqrt{16.25})(1)}{\sqrt{16.5}} = 0.9923$$

COMPLEX POWER –

consider RL load:

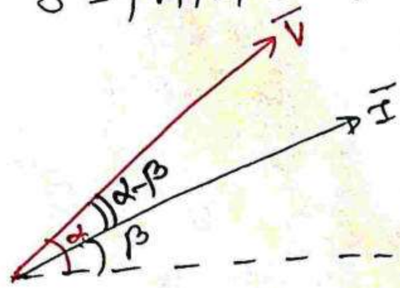
$\bar{V}_{rms} = |V| \angle \alpha$

$I_{rms} = |I| \angle \beta$

$S = \bar{V} \cdot \bar{I}$

$= |V| \angle \alpha \cdot |I| \angle \beta$

$S = |V||I| \angle \alpha + \beta$



$S = |V||I| \angle \alpha - \beta$

$S = \bar{V}_{rms} \bar{I}_{rms}^*$

- Complex power, $S = \bar{V}\bar{I}^*$
- Apparent power $= |S|$
- $S = \bar{V} I^*$

$= |V||I| \angle \alpha - \beta$

Let $\alpha - \beta = \theta$

$= |V||I| e^{j\theta}$

$S = |V||I| [\cos\theta + j\sin\theta]$

$S = |V||I|\cos\theta + j|V||I|\sin\theta$

$S = P + jQ$

- $\bar{V} = |V| \angle \theta_v$, $\bar{I} = |I| \angle \theta_i$

$P = |V||I| \cos(\theta_v - \theta_i)$

$= Re\{|V||I| e^{j(\theta_v - \theta_i)}\}$

$P = Re[|V| e^{j\theta_v} \cdot |I| e^{j\theta_i}]$

$P = Re[|V| \angle \theta_v \cdot |I| \angle -\theta_i]$

- complex Power –

$S = |V| \angle \theta_v \cdot |I| \angle -\theta_i$

$S = \bar{V} \bar{I}^*$

Q. Voltage across a load is

$v(t) = 60\cos(\omega t - 10^\circ)$ V & current through the load in the direction of voltage drop is

$i(t) = 1.5\cos(\omega t + 50^\circ)$ A.

Find, the complex power, apparent power, real power & reactive power.

Solⁿ $\bar{V} = \frac{60}{\sqrt{2}} \angle -10^\circ$ V

$\bar{I} = \frac{1.5}{\sqrt{2}} \angle 50^\circ$ A

- Complex power (S)

$S = \bar{V}\bar{I}^*$

$= \frac{60}{\sqrt{2}} \angle -10^\circ \cdot \frac{1.5}{\sqrt{2}} \angle -50^\circ$

$S = 45\angle -60^\circ$ VA

- Apparent power =

$|S| = 45$ VA

- $S = 45[\cos 60^\circ - j\sin 60^\circ]$

$S = 45\left[\frac{1}{2} - j\frac{\sqrt{3}}{2}\right]$

$S = 22.5 - j38.97$

$P = 22.5$ W

$Q = -38.97$ VAR.